Do now as a warm-up:

1. Devise your own definition of a limit (or look it up). Be able to explain to the class what you know about limits.

2. What methods do you know for finding limits? How would you begin finding a limit, if you were asked to find one? How many methods can you list?

1.2 Finding Limits Graphically and Numerically

GNAW on limits:

Graphically: examine a graph

Numerically: make a table of values

Analytically/Algebraically:

- plug in, if possible
- if you get the indeterminate form 0/0, try:
  - factor and reduce
  - square root? multiply top and bottom by conjugate
- try re-writing the function, maybe it's piecewise

Written: use correct math notation, be precise and concise

G&N are great for when a graphing calculator is permitted!

A&W are important for when a graphing calculator is not permitted!

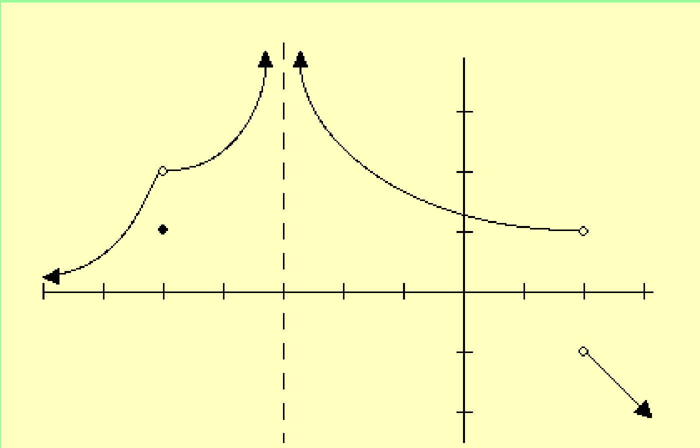
Make a table of values to find each limit. If the limit does not exist, what "interesting" thing happens to the graph when x=3?

$$\lim_{x \to 3} \frac{x^2 - 7x + 12}{x - 3}$$

$$\lim_{x \to 3} \frac{1}{x^2 - 9}$$

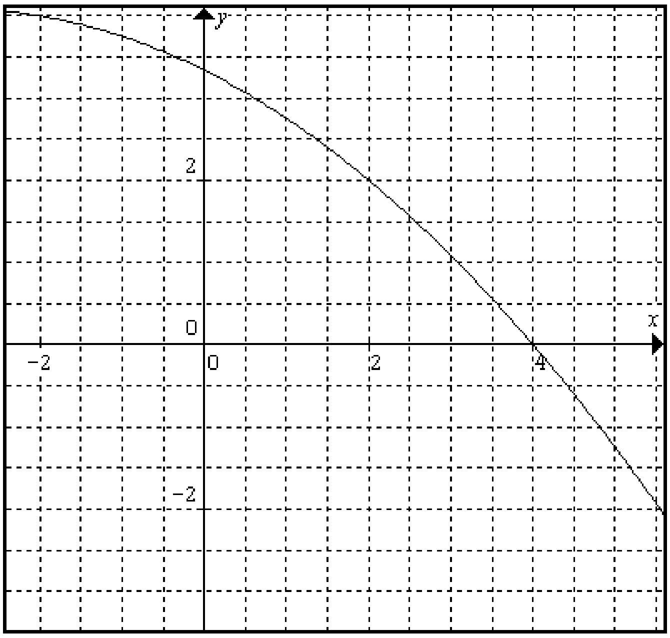
$$\lim_{x \to 3} \frac{|x - 3|}{x - 3}$$

Visual Limits

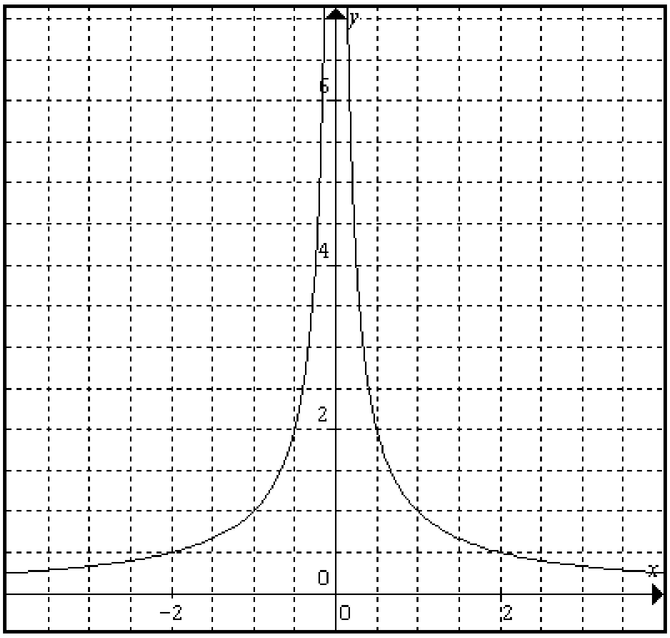


1. $f(2)$
2. $\lim_{x\to 2^+} f(x)$
3. $\lim_{x\to 2^-} f(x)$
4. $\lim_{x\to 2} f(x)$
5. $f(-5)$
6. $\lim_{x\to -5^+} f(x)$
7. $\lim_{x\to -5^-} f(x)$
8. $\lim_{x\to -5} f(x)$
9. $\lim_{x\to -3} f(x)$
10. $\lim_{x\to -\infty} f(x)$
11. $\lim_{x\to \infty} f(x)$
12. $\lim_{x\to c} f(x)$ for $c \neq -5, -3, 2.$

ex. Find $\lim_{x \to 4} f(x)$ graphically.



ex. Find $\lim_{x \to 0} f(x)$ graphically.



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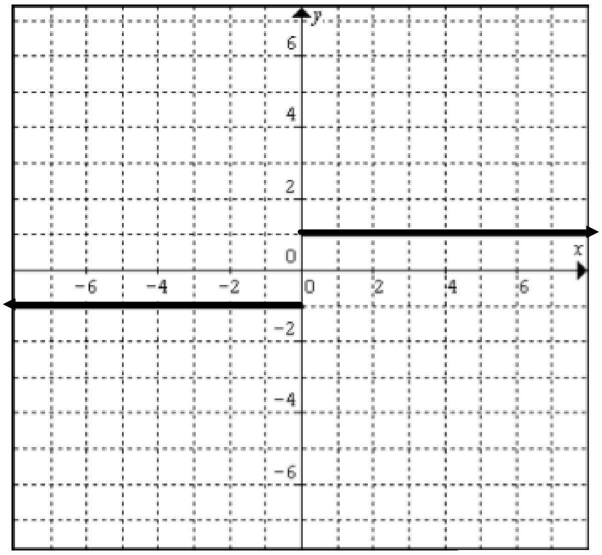

ex. Find $\lim_{x\to 0^+} \frac{|x|}{x}$

ex. Find

$\lim_{x\to 0^-} \frac{|x|}{x}$

ex. Find $\lim_{x\to 0} \frac{|x|}{x}$

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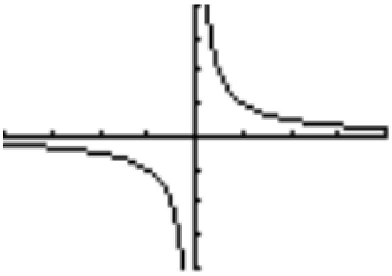
ex. Find

$$\lim_{x \to 0^-} \frac{|x|}{x}$$

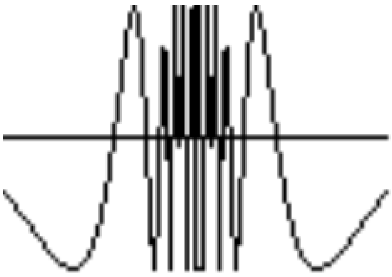


ex. Find $\lim_{x \to 0^+} \frac{|x|}{x}$

ex. Find $\lim_{x \to 0} \frac{|x|}{x}$

Another reason a limit doesn't exist:
the left limit ≠ the right limit

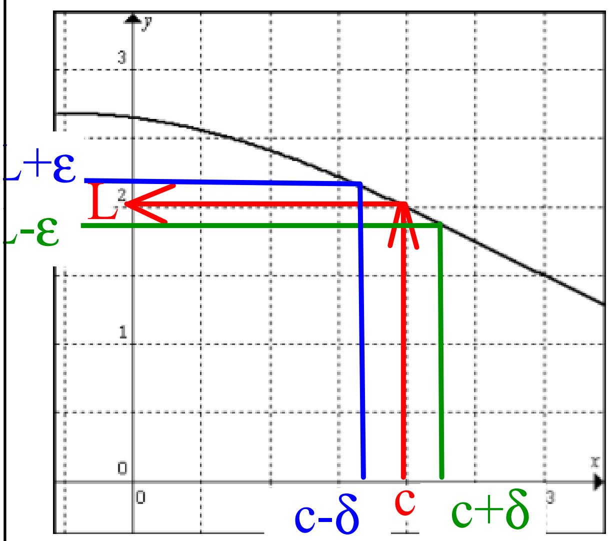
ex. $\lim_{x \to 0} \frac{x^3 + x}{|x|}$



ex. Find $\lim_{x \to 0} \cos\left(\frac{1}{x}\right)$

info

http://www.slu.edu/classes/maymk/Applets/EpsilonDelta.html



The limit of a function is the height (y value) that we would ***expect*** when we get ***close to*** a particular x value.

Here, c, L, δ, and ε are all real numbers.

When x gets close to c (say δ above or below c), then we get a y value close to L (no more than ε above or below L).

Defn. Limit:
f is a function defined on an open interval around a number c (but not necessarily at c)
$L \in \mathbf{R}$, $\delta \in \mathbf{R}$, and $\varepsilon \in \mathbf{R}$,
note also that $\varepsilon > 0$ and $\delta > 0$ then,

$$\lim_{x \to c} f(x) = L$$

means that if x is within δ of c ($|x-c| < \delta$),
then f(x) will be within ε of L ($|f(x)-L| < \varepsilon$).

ex. Given $\lim_{x \to 3}(2x-5)=1$ Find δ given ε=.01.

L

c

Remember from the definition of a limit
that $|f(x)-L| < \varepsilon$ whenever $|x-c| < \delta$.

We start here in such a problem...

... and derive this part.

ex. Assuming $\lim_{x \to 1}(6x+4)=10$ and that ε=.001, find δ.

ex. Assuming $\lim_{x \to 2}(3x-2)=4$

what δ would be chosen for any given ε?