

Do now as a warm-up:

1. Devise your own definition of a limit (or look it up). Be able to explain to the class what you know about limits.

2. What methods do you know for finding limits? How would you begin finding a limit, if you were asked to find one? How many methods can you list?

**1.2 Finding Limits Graphically and Numerically**

GNAW on limits:

Graphically: examine a graph

Numerically: make a table of values

Analytically/Algebraically:

plug in, if possible

if you get the indeterminate form  $0/0$ , try:

factor and reduce

square root? multiply top and bottom by conjugate

try re-writing the function, maybe it's piecewise

Written: use correct math notation, be precise and concise

G&N are great for  
when a graphing  
calculator is permitted!

A&W are important for  
when a graphing  
calculator is not permitted!

Make a table of values to find each limit. If the limit does not exist, what "interesting" thing happens to the graph when  $x=3$ ?

$$\lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{x - 3} = -1$$

x	2.9	2.999	3.001	3.1
y	-1.1	-1.001	-0.999	-0.9

hole at (3, -1)

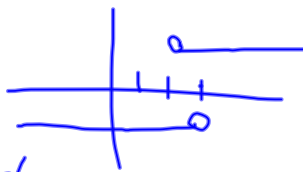
$$\lim_{x \rightarrow 3} \frac{1}{x^2 - 9}$$

does not exist

x	2.9	2.999	3.001	3.1
y	-1.695	-166.694	166.639	1.639

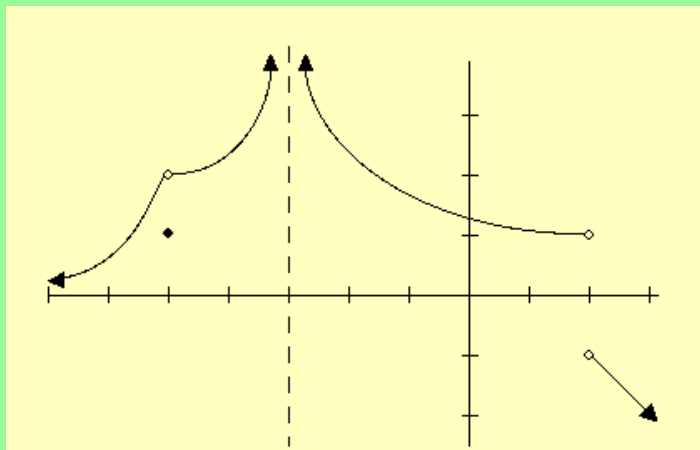
$$\lim_{x \rightarrow 3} \frac{|x - 3|}{x - 3}$$

does not exist



x	2.9	2.999	3.001	3.1
y	-1	-1	1	1

## Visual Limits



1.  $f(2)$

2.  $\lim_{x \rightarrow 2^+} f(x)$

3.  $\lim_{x \rightarrow 2^-} f(x)$

4.  $\lim_{x \rightarrow 2} f(x)$

5.  $f(-5)$

6.  $\lim_{x \rightarrow -5^+} f(x)$

7.  $\lim_{x \rightarrow -5^-} f(x)$

8.  $\lim_{x \rightarrow -5} f(x)$

9.  $\lim_{x \rightarrow -3} f(x) = \infty$

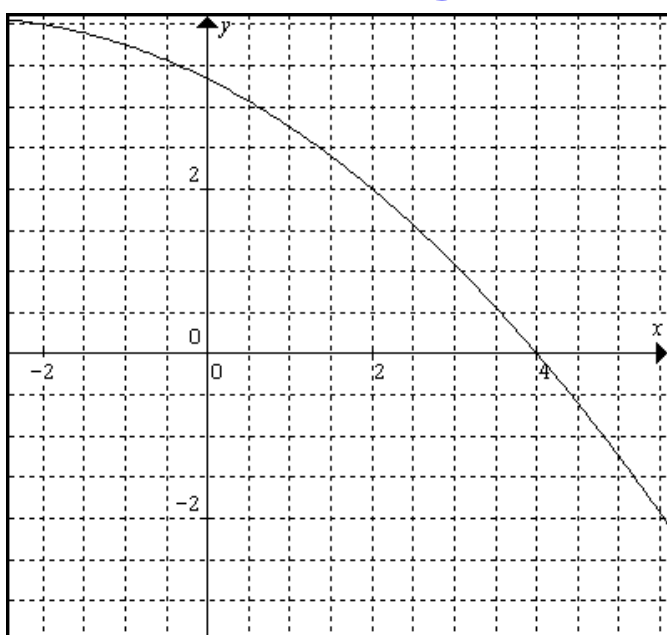
10.  $\lim_{x \rightarrow -\infty} f(x)$

11.  $\lim_{x \rightarrow \infty} f(x)$

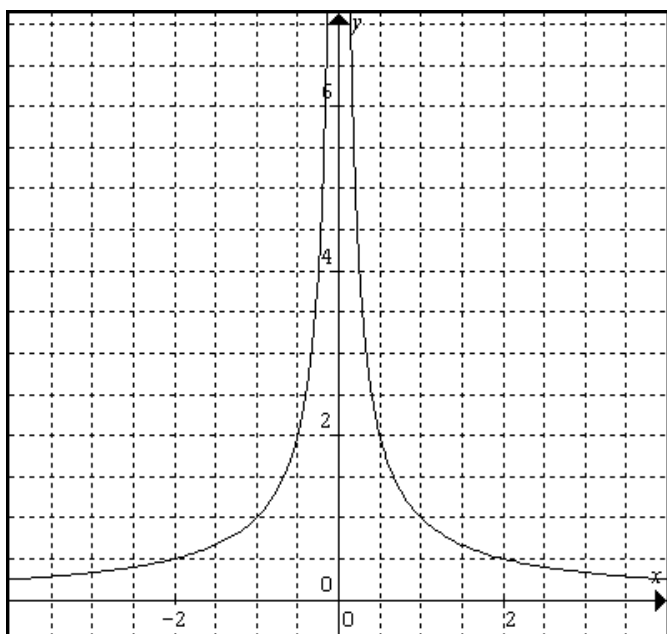
12.  $\lim_{x \rightarrow c} f(x)$  for  $c \neq -5, -3, 2$ .

ex. Find  $\lim_{x \rightarrow 4} f(x)$  graphically.

$$= 0$$



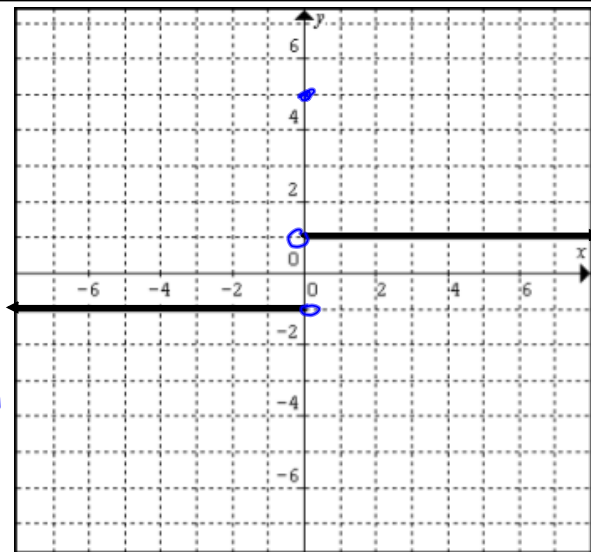
ex. Find  $\lim_{x \rightarrow 0} f(x) = \infty$  graphically.  
DNE



One reason a limit  
doesn't exist:  
it goes to  $\infty$  or  $-\infty$

ex. Find

$$\lim_{x \rightarrow 0^-} \frac{|x|}{x} = -1$$



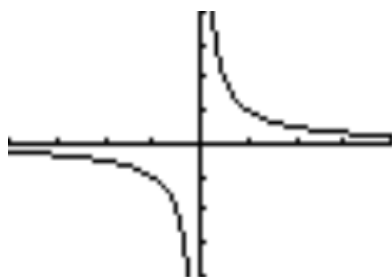
ex. Find  $\lim_{x \rightarrow 0^+} \frac{|x|}{x} = 1$

ex. Find  $\lim_{x \rightarrow 0} \frac{|x|}{x}$  DNE

Another reason a  
limit doesn't exist:  
the left limit  $\neq$  the right limit

ex. Find

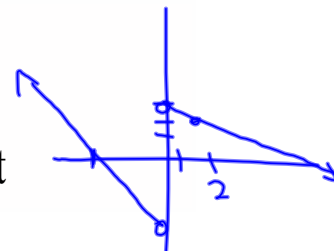
$$\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty \quad \text{DNE}$$



$$\text{ex. Find } \lim_{x \rightarrow 0^+} \frac{1}{x} = \infty \quad \text{DNE}$$

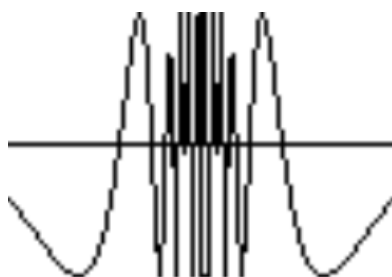
$$\text{ex. Find } \lim_{x \rightarrow 0} \frac{1}{x} \quad \text{DNE}$$

Another reason a  
limit doesn't exist:  
the left limit  $\neq$  the right limit



$$\text{ex. } \lim_{x \rightarrow 0} \frac{x^3 + x}{|x|} \quad \text{DNE because } \lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$



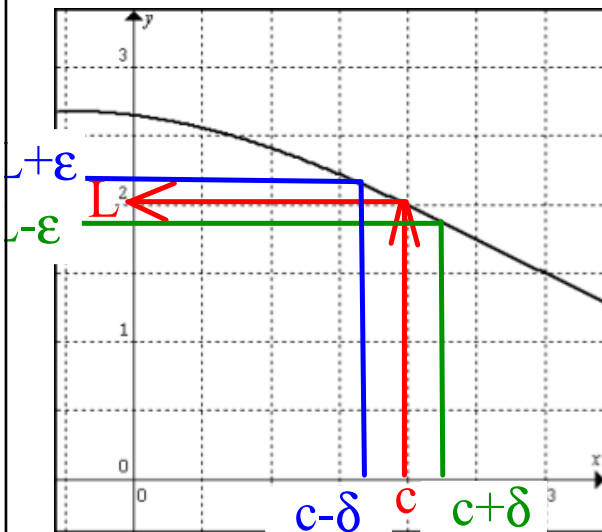


ex. Find  $\lim_{x \rightarrow 0} \cos\left(\frac{1}{x}\right)$  DNE

$x$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{10}$	$\frac{1}{100} \rightarrow 0$
$\frac{1}{x}$	1	2	3	10	100 $\rightarrow \infty$

A third reason a limit  
doesn't exist:  
the function oscillates

<http://www.slu.edu/classes/maymk/Applets/EpsilonDelta.html>



The limit of a function is the height (y value) that we would expect when we get close to a particular x value.

Here,  $c$ ,  $L$ ,  $\delta$ , and  $\epsilon$  are all real numbers.

When  $x$  gets **close to  $c$**  (say  $\delta$  **above** or **below**  $c$ ), then we get a **y value close to  $L$**  (no more than  $\epsilon$  **above** or **below**  $L$ ).

Defn. Limit:

$f$  is a function defined on an open interval around a number  $c$  (but not necessarily at  $c$ )

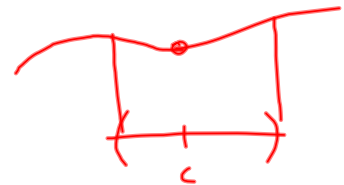
$L \in \mathbf{R}$ ,  $\delta \in \mathbf{R}$ , and  $\varepsilon \in \mathbf{R}$ ,

note also that  $\varepsilon > 0$  and  $\delta > 0$  then,

$$\lim_{x \rightarrow c} f(x) = L$$

means that if  $x$  is within  $\delta$  of  $c$  ( $|x-c| < \delta$ ),

then  $f(x)$  will be within  $\varepsilon$  of  $L$  ( $|f(x)-L| < \varepsilon$ ).



ex. Given  $\lim_{x \rightarrow 3} (2x - 5) = 1$  Find  $\delta$  given  $\epsilon = .01$ .

$x \rightarrow 3$

$L$

$c$

D	R
x	y
c	L
$\delta$	$\epsilon$

Remember from the definition of a limit that  $|f(x) - L| < \epsilon$  whenever  $|x - c| < \delta$ .

We start here ... and derive this part in such a problem...

$$|2x - 5 - 1| < .01$$

$$|2x - 6| < .01$$

$$2|x - 3| < .01$$

$$|x - 3| < \frac{.01}{2} = .005 = \delta$$

$$1 \pm .01$$

$$y \in (.99, 1.01)$$

$$x \in 3 \pm .005$$

$$(2.995, 3.005)$$

ex. Assuming  $\lim_{x \rightarrow 1} (6x+4) = 10$  and that  $\epsilon = .001$ , find  $\delta$ .

ex. Assuming  $\lim_{x \rightarrow 2} (3x - 2) = 4$   
what  $\delta$  would be chosen for  
any given  $\epsilon$ ?