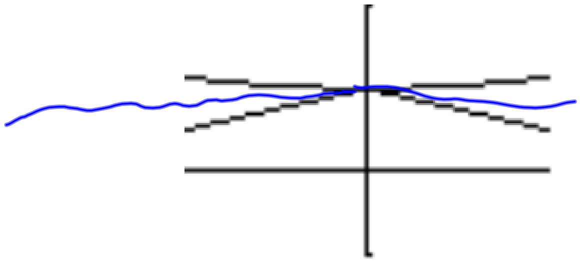
Do now as a warm-up:

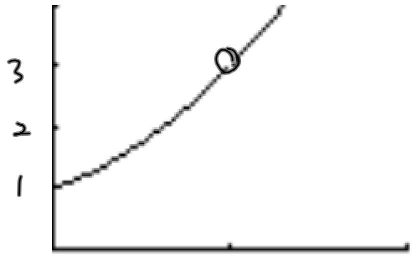


I'm thinking of a function.
Let's call it f(x).
Suppose I only look at x values from -0.5 to 0.5.
For those x values, f(x) is at least as big as 1-|x|
and is no bigger than 1/(cosx).
What does f(x) do near 0?

# 1.3 Evaluating Limits Analytically

GNAW on this:

$$\lim_{x\to 1}\frac{x^3-1}{x-1}$$




| x | .99 | 1 | 1.01 |
|---|---|---|---|
| y | 2.9701 | undef'd | 3.0301 |

$$\lim_{x\to 1}\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}} = \boxed{1+1+1} = 3$$

as $x$ approaches 1, $\frac{x^3-1}{x-1}$ approaches 3.

ex. Evaluate analytically: $\lim_{x \to 0} \frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}$

$$= \lim_{x \to 0} \frac{\cancel{x}^{1} + \cancel{4} - \cancel{4}}{\cancel{x}_{1}(\sqrt{x+4}+2)}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{x+4}+2} = \frac{1}{4}$$

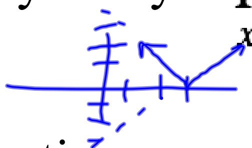
ex. Evaluate analytically: $\lim_{x \to 3} \frac{x-3}{\sqrt{3}-\sqrt{x}} \cdot \frac{\sqrt{3}+\sqrt{x}}{\sqrt{3}+\sqrt{x}}$

$$= \lim_{x \to 3} \frac{\overset{-1}{\cancel{(x-3)}}(\sqrt{3}+\sqrt{x})}{\underset{1}{\cancel{3-x}}} = (-1)(\sqrt{3}+\sqrt{3})$$

$$= -2\sqrt{3}$$

$$\frac{2\sqrt{3}}{-1} =$$

ex. Evaluate analytically: $\lim_{x \to 3} |x - 3| = 0$



Re-write the function as a piecewise defined function.

$$f(x) = |x - 3|$$

$$f(x) = \begin{cases} -x + 3, & x < 3 \\ x - 3, & x \geq 3 \end{cases}$$

ex. Evaluate analytically: $\lim_{x \to 3} \frac{x^2-5x+6}{x-3}$

$$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x-2)}{\cancel{x-3}} = 1$$

ex. Evaluate analytically: $\lim\limits_{x \to -1} \dfrac{\cancel{x^2-1}\,(x+1)(x-1)}{(x+1)^2}$

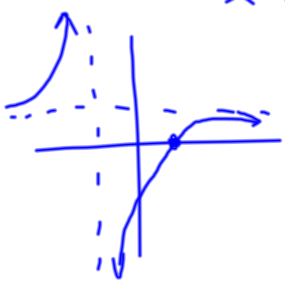
$= \lim\limits_{x \to -1} \dfrac{x-1}{x+1}$

$x = -2, \quad \dfrac{-2-1}{-2+1} = \dfrac{-3}{-1} = 3$

$x = 0, \quad \dfrac{0-1}{0+1} = -1$

so $\lim\limits_{x \to -1^-} \dfrac{x-1}{x+1} = \infty$

so $\lim\limits_{x \to -1^+} \dfrac{x-1}{x+1} = -\infty$



$\lim\limits_{x \to -1} \dfrac{x^2-1}{(x+1)^2}$ does not exist ex. Evaluate analytically: $\lim_{x \to 2} \frac{x - \sqrt{6-x}}{(x-2)} \cdot \frac{x+\sqrt{6-x}}{x+\sqrt{6-x}}$

$$\lim_{x \to 2} \frac{x^2 - (6-x)}{(x-2)(x+\sqrt{6-x})}$$

$$(x+3)\cancel{(x-2)}$$

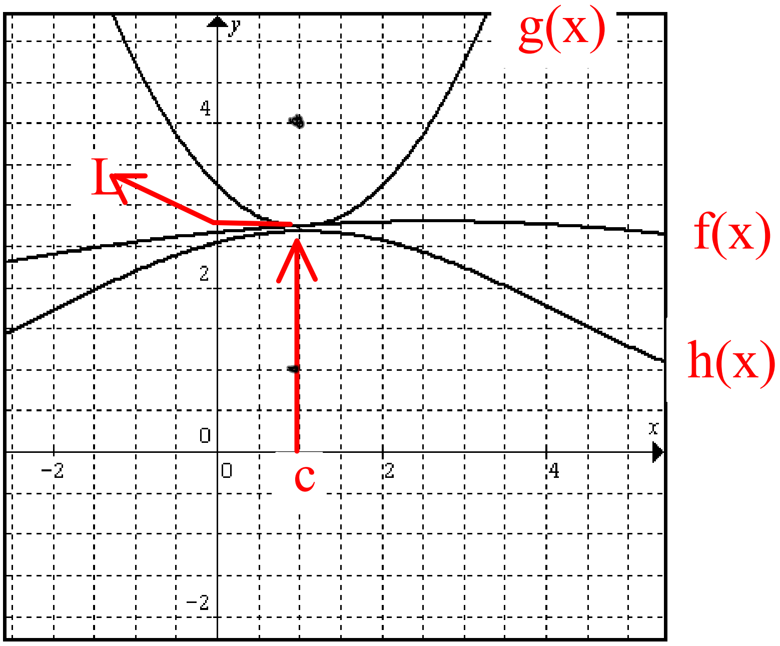
$$\lim_{x \to 2} \frac{x^2 + x - 6}{\cancel{(x-2)}(x+\sqrt{6-x})}$$

$$\lim_{x \to 2} \frac{x+3}{x+\sqrt{6-x}} = \frac{5}{4}$$

Thm. Squeeze Theorem:

If $h(x) \leq f(x) \leq g(x)$ $\forall x \in$ an open interval containing c, except possibly at c, and if $\lim_{x \to c} h(x) = L = \lim_{x \to c} g(x)$

then $\lim_{x \to c} f(x)$ exists and is L.



Thm. $\lim_{x\to 0} \frac{\sin x}{x} = 1$ and $\lim_{x\to 0} \frac{1-\cos x}{x} = 0$

ex. Evaluate analytically: $\lim_{x\to 0} \frac{\sin 5x}{x} \cdot \frac{5}{5}$

$= \lim_{x\to 0} \frac{5 \sin 5x}{5x} = \lim_{x\to 0} \frac{5}{1} \cdot \frac{\sin 5x}{5x} = 5$

ex. Evaluate analytically: $\lim_{x\to 0} \frac{x \sin 2x}{\sin x}$

$= \lim_{x\to 0} \frac{x 2 \cancel{\sin x} \cos x}{\cancel{\sin x}} = 0$