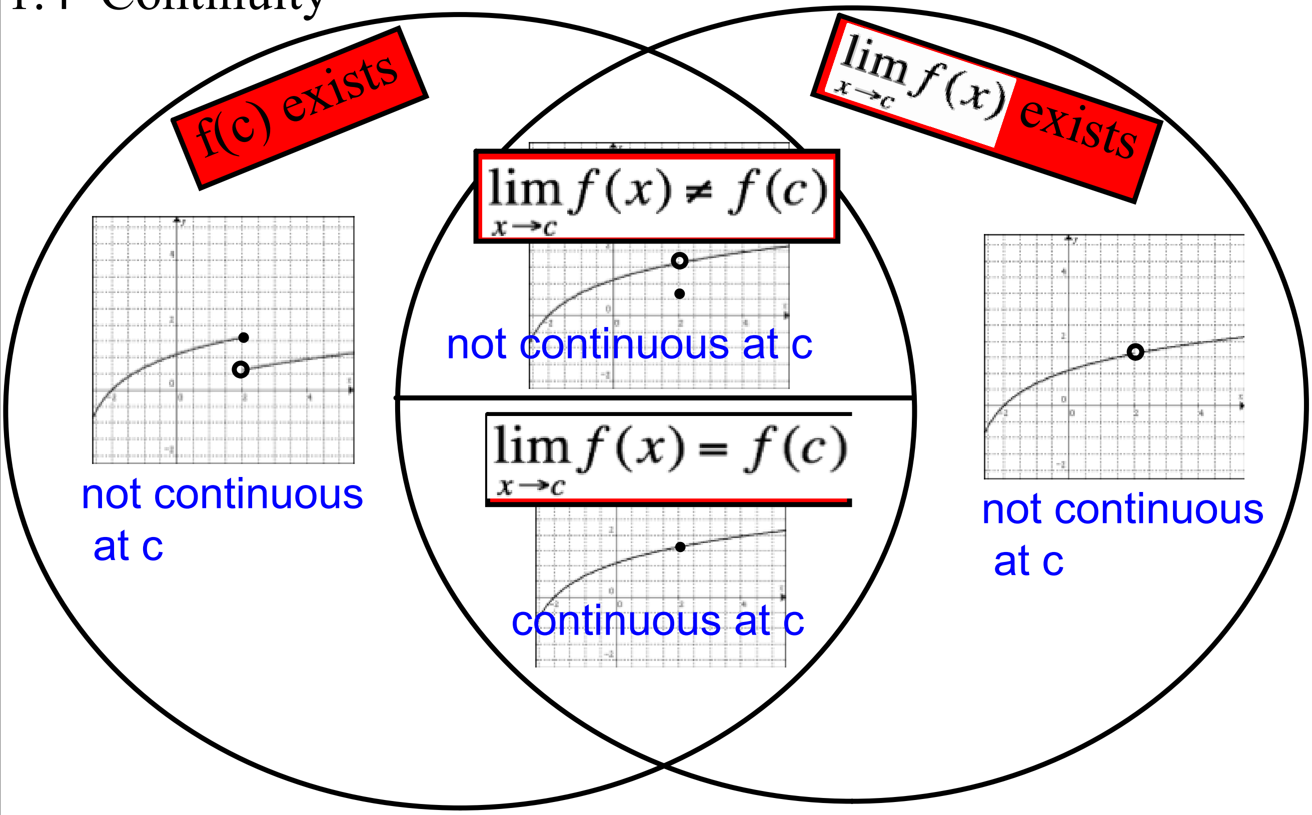
Do now as a warm-up:

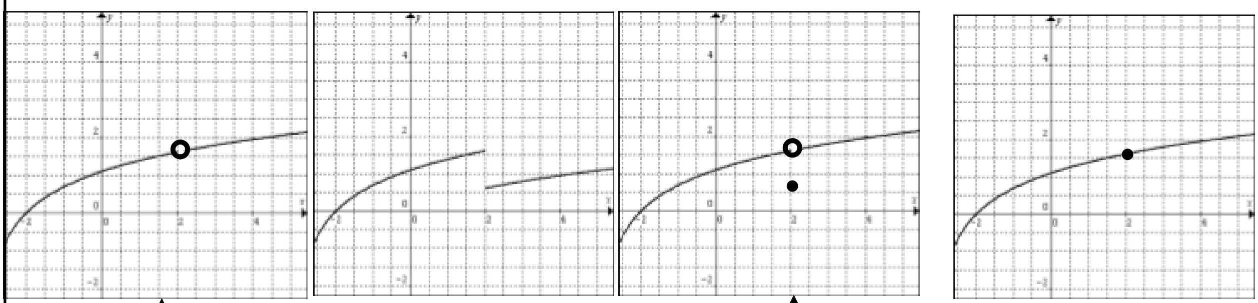
Is there some number a,
such that this limit exists?

$$\lim_{x \to 3} \frac{2x^2 - 3ax + x - a - 1}{x^2 - 2x - 3}$$

If so, find the value of a
and find the limit.
If not, explain why not.

1.4 Continuity





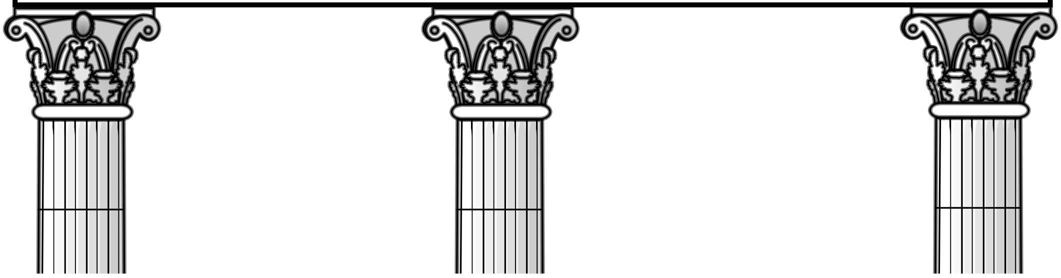
f undefined at c
$\lim_{x\to c} f(x)$ exists
not continuous at c

f defined at c
$\lim_{x\to c} f(x)$ D.N.E.
not continuous at c

f defined at c
$\lim_{x\to c} f(x)$ exists
but $\lim_{x\to c} f(x) \neq$ f(c)
not continuous at c

f defined at c
$\lim_{x\to c} f(x)$ exists
and they're =
continuous at c

Defn. A function f is continuous at c if all these hold:



f(c) is defined $\lim_{x \to c} f(x)$ exists $\lim_{x \to c} f(x) = f(c)$

Defn. A function f is continuous on an open interval (a,b) if it is continuous at every point in (a,b).

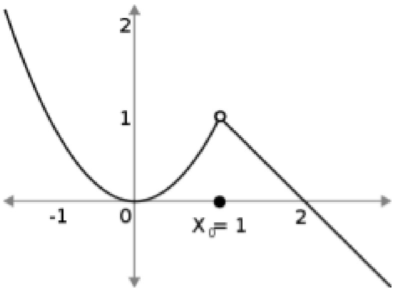
Classifying Discontinuities

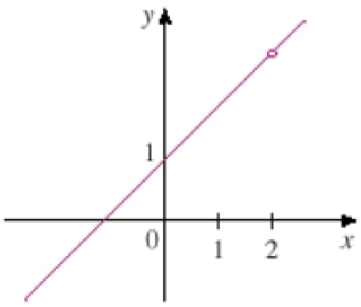
Removable or point
(Holes)
2 sided limit exists

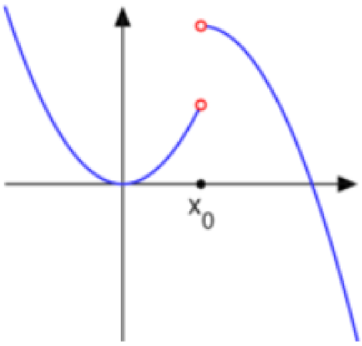
Essential or Non-removable

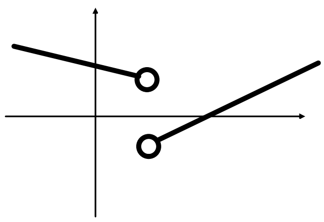
Jump
1 sided limits exist

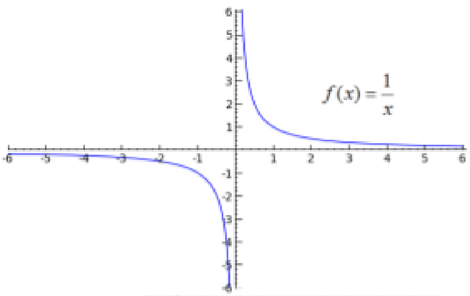
Infinite
(vertical asymptotes)
at least one of the
1 sided limits don't exist

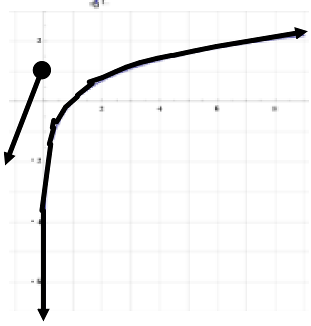












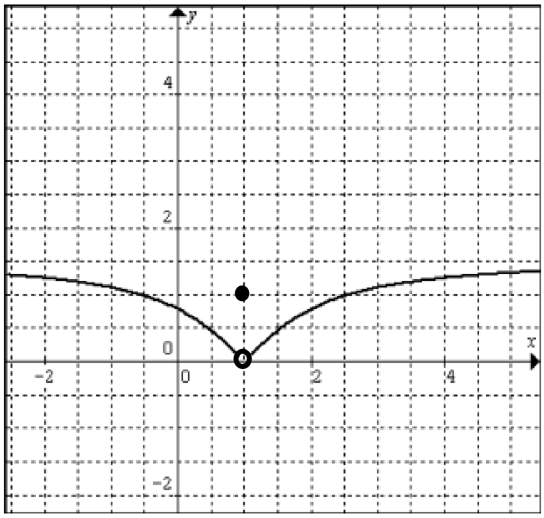
ex. Evaluate:

$$\lim_{x \to 1^-} f(x)$$

$$\lim_{x \to 1^+} f(x)$$

$$\lim_{x \to 1} f(x)$$

f(1)



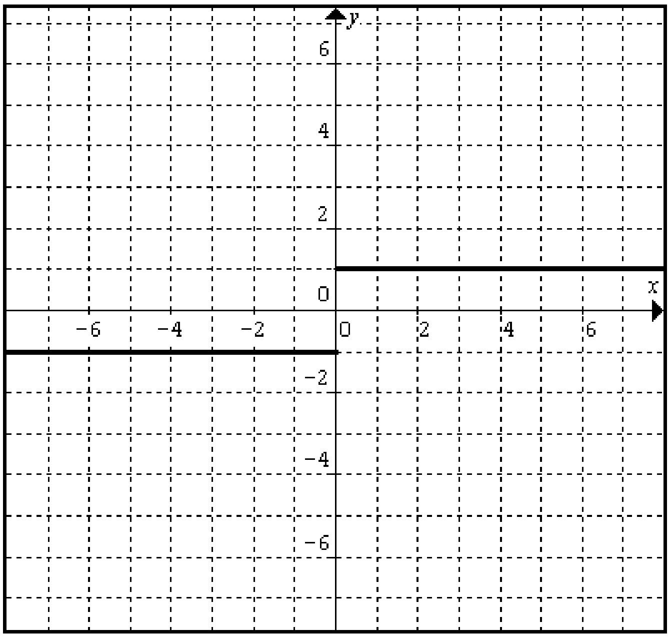
Is f continuous at x=1?

ex. Evaluate these limits:

$\lim_{x \to 0-} f(x)$

$\lim_{x \to 0+} f(x)$

$\lim_{x \to 0} f(x)$



f(0)

Is f continuous at x=0?

Thm.

given:

f is a function $c \in \mathbb{R}$ and $L \in \mathbb{R}$

$\lim_{x \to c} f(x) = L$ $\Longleftrightarrow$ $\lim_{x \to c^-} f(x) = L$ and $\lim_{x \to c^+} f(x) = L$

Thm.

f is continuous on (a,b)

$\lim_{x \to a^+} f(x) = f(a)$

$\lim_{x \to b^-} f(x) = f(b)$

$\Longleftrightarrow$ f is continuous on [a,b] ex. Find a and b so that f(x) is everywhere continuous.

$$f(x) = \begin{cases} \frac{1}{8}ax^3 - bx + 3, & x > 2 \\ 2x - \frac{1}{2}x^2, & x = 2 \\ \frac{1}{4}ax^3 - bx - 5, & x < 2 \end{cases}$$

Discuss the continuity of: $f(x) = \begin{cases} 0, & \text{x is rational} \\ 1, & \text{x is irrational} \end{cases}$

Thm.

f(x) and g(x) are continuous at x=c

$b \in \mathbf{R}$

These are continuous at c

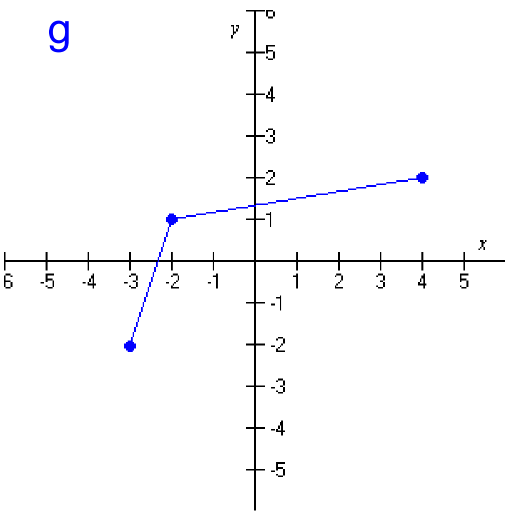
(b)(f(x)) f+g f-g

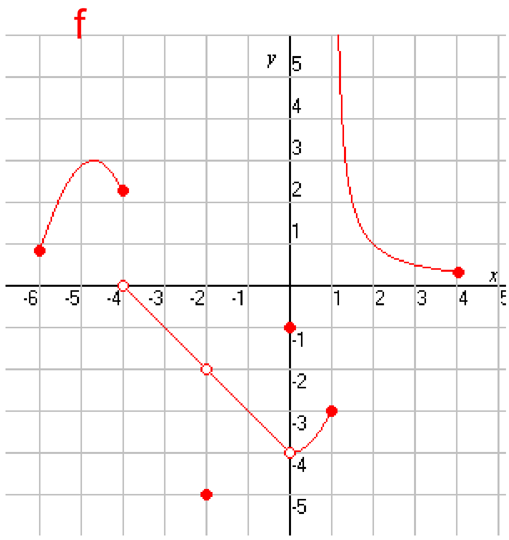
fg f/g (as long as $g(c) \neq 0$)

Thm. Continuity of a composite

g is continuous at c
f is continuous at g(c)

→ f(g(x)) is continuous at c





Is f(g(x)) cont's at...
0?
-7/3?
-2?
-5?
-3?

ex. Discuss the continuity of the composite function f(g(x)) if...

$f(x) = \frac{1}{\sqrt{x}}$ and $g(x) = x - 1$

$f(x) = x + 4$ and $g(x) = \frac{1}{x - 6}$

ex. Find the domain for each function.
Where is each function not cont's?
What kind of discontinuity is this?

$y=\sqrt{x+2}$

$y=\frac{1}{x^2+5x-6}$

ex. Find the domain.
Where is the function not cont's?
What kind of discontinuity is this?

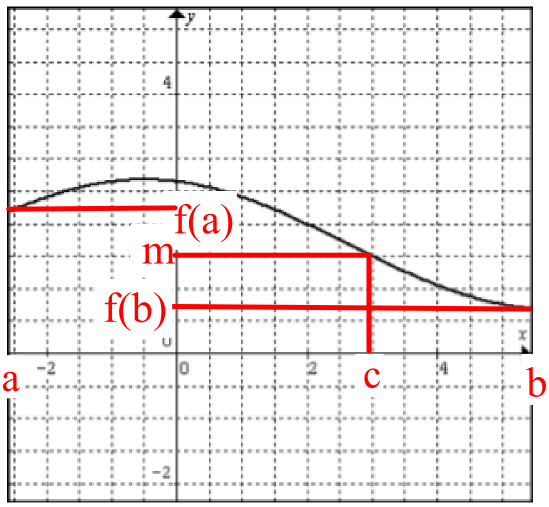
$y = \tan x$

Thm. The Intermediate Value Theorem (IVT)

f is continuous on [a,b] $m \in R$ $f(a)<m<f(b)$	→	$\exists c \in [a,b]$ where $f(c)=m$

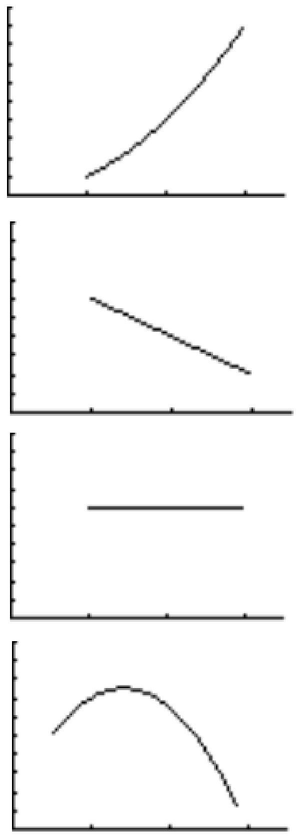
This is an existence theorem. It says something exists, but not where.

Try it!
Draw a cont's function between (a,f(a)) and (b,f(b)).
Pick any y value between f(a) and f(b).
A horizontal line through that y value has to cross your function.



http://www.calculusapplets.com/ivt.html

The IVT holds for these:

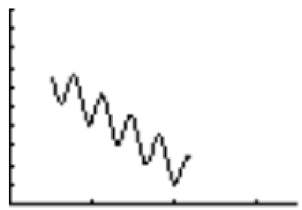


f is continuous on [a,b]

$m \in R$

$f(a) < m < f(b)$

The IVT even holds for this:



It says there's at least one c value that works. There could be many!

But, if f is not continuous on [a,b], the theorem doesn't tell us anything.

Apply the IVT, if possible, on [0,5] so that f(c)= 11 for the function $f(x)=x^2+x-1$

Apply the IVT, if possible, on $[0,3\pi/2]$ for $y=\sin x$ so that $f(c)=0$.