Do now as a warm-up:

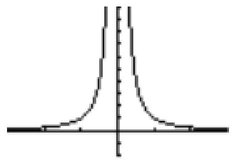
What do you remember about asymptotes?

GNAW on this--
Try to come up with some fact about asymptotes to go with each "dialect" of calculus:

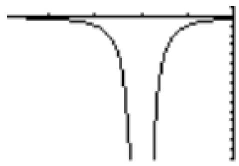
Graphically
Numerically
Analytically/Algebraically
Written

1.5 Infinite limits

Defn. <u>Infinite limits</u>
Let f be a function defined at every real number except possibly at c, then $\lim_{x\to c} f(x)=\infty$
means that for each m>0, $\exists \delta>0$
such that f(x)>m whenever 0<|x-c|<δ.



Similarly, for $\lim_{x\to c} f(x)=-\infty$,
for each N<0, $\exists \delta>0$
such that f(x)<N whenever 0<|x-c|<δ.



ex. Evaluate: $\lim_{x \to 1^+} \frac{x+1}{x-1}$

ex. Evaluate: $\lim_{x \to 1} \frac{1}{(x-1)^2}$

Defn. If f(x) approaches ±∞ as x approaches c from the right or left, then the line x=c is a vertical asymptote.

Thm.

h(x)=f(x)/g(x) f(c)≠0 g(c)=0	→	x=c is a vertical asymptote for h

$h(c) = \frac{nonzero}{zero}$ x=c is a vertical asymptote	$h(c) = \frac{zero}{zero}$ x=c is the location of a hole

ex. Find any vertical asymptotes or removable discontinuities:

$$f(x) = \frac{x-2}{x^2 - x - 2}$$

$$f(x) = \frac{x^2+1}{(x^2+1)(\sin x)}$$

$$f(x) = \frac{x^2-1}{(x^2-1)(\sin x)}$$

Properties of limits:

$\lim_{x \to 0} c = c \longrightarrow \lim_{x \to 0} 4 =$

$\lim_{x \to c} x = c \longrightarrow \lim_{x \to 5} x =$

Thm. $\lim_{x \to a} f(x) = b \longrightarrow \lim_{x \to a} [c \bullet f(x)] = c \bullet b$

so, $\lim_{x \to 5} f(x) = 3 \longrightarrow \lim_{x \to 5} [6 \bullet f(x)] =$

More properties of limits:

Suppose

$$\lim_{x \to 5} f(x) = 3$$

$$\lim_{x \to 5} g(x) = 2$$

What do you expect these to be?

$$\lim_{x \to 5}\left[f(x) + g(x)\right] =$$

$$\lim_{x \to 5}\left[f(x) - g(x)\right] =$$

$$\lim_{x \to 5}\left[f(x) \bullet g(x)\right] =$$

More properties of limits:

Suppose

$$\lim_{x \to 5} f(x) = 3$$

$$\lim_{x \to 5} g(x) = 2$$

What do you expect this to be?

$$\lim_{x \to 5} \left[\frac{f(x)}{g(x)} \right] =$$

In general, what if the limit of a fraction like this gives...

$\frac{c}{0}$? $\frac{0}{c}$? $\frac{0}{0}$?

More properties of limits:

Suppose

$$\lim_{x \to 5} f(x) = 3$$

$$\lim_{x \to 5} g(x) = 2$$

$$\lim_{x \to 2} f(x) = 19$$

What do you expect these to be?

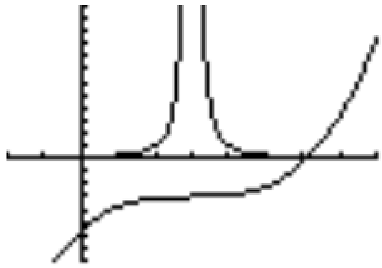
$$\lim_{x \to 5} [g(x)]^6 =$$

$$\lim_{x \to 5} \sqrt{g(x)} =$$

$$\lim_{x \to 5} f\left(g(x)\right) =$$

Properties of limits:

Let $\lim_{x \to c} f(x) = \infty$ and $\lim_{x \to c} g(x) = L$



1. $\lim_{x \to c} \left[f(x) \pm g(x) \right] = \infty$

2. $\lim_{x \to c} \left[f(x) \bullet g(x) \right] = \infty$ if L>0

   $\lim_{x \to c} \left[f(x) \bullet g(x) \right] = -\infty$ if L<0

3. $\lim_{x \to c} \left[g(x) / f(x) \right] = L/\infty = 0$

Thm. $\lim_{x \to 0} \frac{c}{x^n} = \infty$

Ex. Evaluate:

$$\lim_{x\to 0} \frac{1}{x^2}$$

$$\lim_{x\to 0} 1+\frac{1}{x^2}$$

$$\lim_{x\to 0}\left(\frac{1}{x^4}\right)(-4+x)$$

$$\lim_{x\to\infty}\left(\frac{\left(\frac{5x-3}{2x+3}\right)}{\left(x^4\right)}\right)$$

Ex. Identify all vertical asymptotes of
$f(x)=\cot x$.