

Do now as a warm-up:

What do you remember about asymptotes?

GNAW on this--

Try to come up with some fact about asymptotes to go with each "dialect" of calculus:

Graphically

Numerically

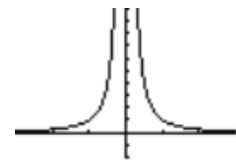
Analytically/Algebraically

Written

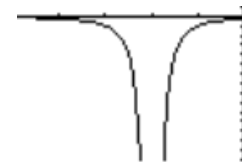
1.5 Infinite limits

Defn. Infinite limits

Let f be a function defined at every real number except possibly at c , then $\lim_{x \rightarrow c} f(x) = \infty$ means that for each $m > 0$, $\exists \delta > 0$ such that $f(x) > m$ whenever $0 < |x - c| < \delta$.



Similarly, for $\lim_{x \rightarrow c} f(x) = -\infty$, for each $N < 0$, $\exists \delta > 0$ such that $f(x) < N$ whenever $0 < |x - c| < \delta$.



ex. Evaluate: $\lim_{x \rightarrow 1^+} \frac{x+1}{x-1} = \infty$

x	1.1	1.01
$\frac{x+1}{x-1}$	$\frac{2.1}{.1} = 21$	$\frac{2.01}{.01} = 201$

ex. Evaluate: $\lim_{x \rightarrow 1} \frac{1}{(x-1)^2} = \infty$

x	1.1	1.01
$\frac{1}{(x-1)^2}$	$\frac{1}{(.1)^2}$ 100	$\frac{1}{(.01)^2}$ 10000

Defn. If $f(x)$ approaches $\pm\infty$ as x approaches c from the right or left, then the line $x=c$ is a vertical asymptote.

Thm.

$$\begin{aligned} h(x) &= f(x)/g(x) \\ f(c) &\neq 0 \\ g(c) &= 0 \end{aligned}$$

$x=c$ is a vertical asymptote for h

$$h(c) = \frac{\textit{nonzero}}{\textit{zero}}$$

$x=c$ is a vertical asymptote

$$h(c) = \frac{\textit{zero}}{\textit{zero}}$$

$x=c$ is the location of a hole

ex. Find any vertical asymptotes or removable discontinuities:

$$f(x) = \frac{x-2}{x^2-x-2}$$

$$(x-2)(x+1)$$

$$x-2=0 \rightarrow x=2 \text{ hole}$$

$$x+1=0 \rightarrow x=-1 \text{ VA}$$

$$f(x) = \frac{x^2+1}{(x^2+1)(\sin x)}$$

VA when $\sin x = 0$, so at

$$x = \dots, -2\pi, -\pi, 0, \pi, 2\pi, \dots$$

$$x = k\pi \text{ where } k \in \mathbb{Z}$$

$$x^2+1=0$$

$$x^2=-1$$

$$f(x) = \frac{x^2-1}{(x^2-1)(\sin x)}$$

$$(x+1)(x-1)$$

holes at $x = \pm 1$ & VA at $x = k\pi$ where $k \in \mathbb{Z}$

Properties of limits:

$$\lim_{x \rightarrow 0} c = c \quad \longrightarrow \quad \lim_{x \rightarrow 0} 4 = 4$$

$$\lim_{x \rightarrow c} x = c \quad \longrightarrow \quad \lim_{x \rightarrow 5} x = 5$$

$$\text{Thm. } \lim_{x \rightarrow a} f(x) = b \quad \longrightarrow \quad \lim_{x \rightarrow a} [c \cdot f(x)] = c \cdot b$$

$$\text{so, } \lim_{x \rightarrow 5} f(x) = 3 \quad \longrightarrow \quad \lim_{x \rightarrow 5} [6 \cdot f(x)] = 6 \cdot 3 \\ = 18$$

More properties of limits:

Suppose

$$\lim_{x \rightarrow 5} f(x) = 3$$

$$\lim_{x \rightarrow 5} g(x) = 2$$

What do you expect these to be?

$$\lim_{x \rightarrow 5} [f(x) + g(x)] = 3 + 2 = 5$$

$$\lim_{x \rightarrow 5} [f(x) - g(x)] = 3 - 2 = 1$$

$$\lim_{x \rightarrow 5} [f(x) \cdot g(x)] = 3 \cdot 2 = 6$$

More properties of limits:

Suppose

$$\lim_{x \rightarrow 5} f(x) = 3$$

$$\lim_{x \rightarrow 5} g(x) = 2$$

What do you expect this to be?

$$\lim_{x \rightarrow 5} \left[\frac{f(x)}{g(x)} \right] = \frac{3}{2}$$

In general, what if the limit of a fraction like this gives...

$$\frac{c}{0} ? \quad \text{limit DNE}$$

$$\frac{0}{c} ? \quad \text{limit} = 0$$

$$\frac{0}{0} ? \quad \text{limit should exist (graph has a hole).}$$

More properties of limits:

Suppose

$$\lim_{x \rightarrow 5} f(x) = 3$$

$$\lim_{x \rightarrow 5} g(x) = 2$$

$$\lim_{x \rightarrow 2} f(x) = 19$$

What do you expect these to be?

$$\lim_{x \rightarrow 5} [g(x)]^6 = 2^6 = 64$$

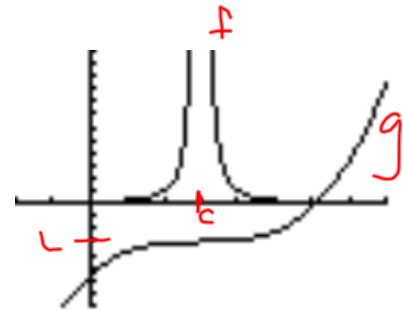
$$\lim_{x \rightarrow 5} \sqrt{g(x)} = \sqrt{2} \text{ or } 1.414$$

$$\lim_{x \rightarrow 5} f(g(x)) = 19$$

Properties of limits:

Let $\lim_{x \rightarrow c} f(x) = \infty$ and $\lim_{x \rightarrow c} g(x) = L$

1. $\lim_{x \rightarrow c} (f(x) \pm g(x)) = \infty$
2. $\lim_{x \rightarrow c} (f(x) \cdot g(x)) = \infty$ if $L > 0$
 $\lim_{x \rightarrow c} (f(x) \cdot g(x)) = -\infty$ if $L < 0$
3. $\lim_{x \rightarrow c} (g(x)/f(x)) = L/\infty = 0$



Thm. $\lim_{x \rightarrow 0} \frac{c}{x^\pi} = \infty$

$$\lim_{x \rightarrow 0} \frac{(\sin x)^2}{x}$$

$$\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \cdot \sin x \right)$$

1 . 0

Ex. Evaluate:

$$\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$$

$$\lim_{x \rightarrow 0} 1 + \frac{1}{x^2} = \infty$$

$$\lim_{x \rightarrow 0} \left(\frac{1}{x^4} \right) (-4 + x) = -\infty$$

$\infty (-4)$

$$\lim_{x \rightarrow \infty} \left(\frac{\frac{5x - 3}{2x + 3}}{x^4} \right) = 0$$

$\frac{5}{2}$
 ∞

Ex. Identify all vertical asymptotes of
 $f(x) = \cot x$.

$$= \frac{\cos x}{\sin x}$$

$$\sin x = 0$$

$$x = \dots - 2\pi, -\pi, 0, \pi, 2\pi, \dots$$

$$x = k\pi, k \in \mathbb{Z}$$