10.1 Conics and Calculus

Recall:

circle $(x-h)^2+(y-k)^2=r^2$

center (h,k)

radius r

parabola $(x-h)^2=4p(y-k)$ $(y-k)^2=4p(x-h)$

up/down left/right

vertex (h,k)

directrix $y=k-p$ $x=h-p$

vertex is p units from the focus

if $4p>0$, open up or right and if $4p<0$, open down or left.

the latus rectum is the chord of length 4p through the focus parallel to the directrix

the tangent line to the parabola at a point P makes equal angles with the line through P that is parallel to the axis and the line $\overleftrightarrow{PF}$.

ellipse

stretched horizontally $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

stretched vertically $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$

center (h,k)

major axis length 2a

minor axis length 2b

foci c units from center on major axis where $c^2=a^2-b^2$

the tangent line to the ellipse at a point P makes equal angles with $\overline{PF_1}$ and $\overline{PF_2}$.

hyperbola $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

opens left/right

opens up/down

center (h.k)

transverse axis connects vertices and has l... $c^2 = a^2 + b^2$

the vertices are a units from the center

the foci are ... $y = k \pm \frac{\sqrt{\#\text{ under } y}}{\sqrt{\#\text{ under } x}}(x-h)$ where

asymptotes

$y = k \pm \frac{b}{a}(x-h)$

or, put another way,

if x is the positive fraction: $y = k \pm \frac{a}{b}(x-h)$

if y is the positive fraction:

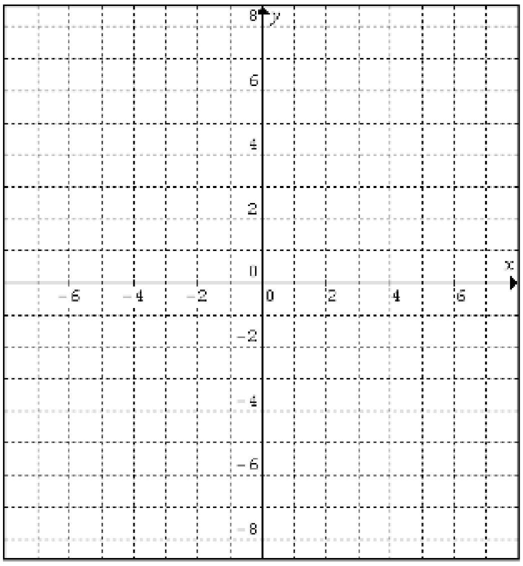
Defn. The eccentricity e of a conic section is $e = \frac{c}{a}$

For any ellipse, $0 < e < 1$.

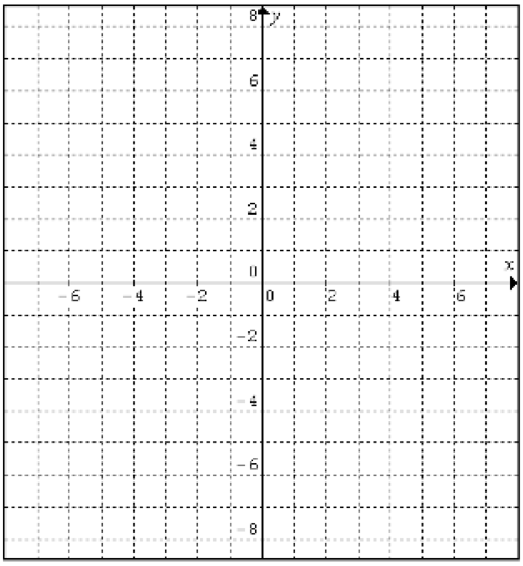
For any parabola, $e = 1$.

For any hyperbola, $e > 1$.

ex. Graph and label $x=y^2-4y+2$



ex. Graph and label $16x^2+9y^2-64x-54y+1=0$



ex. Graph and label $4x^2-9y^2+16x+54y-29=0$