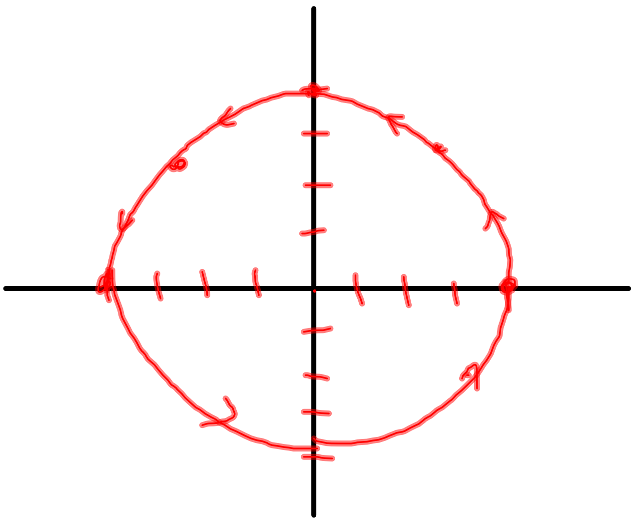
10.2 Plane curves and parametric equations

Defn.

- f and g are cont's functions of t on some interval
- x=f(t) and y=g(t) are called parametric equations
- t is called the parameter.
- the points (x,y) make up a plane curve

ex. Sketch the curve described by $x=4\cos(t)$ and $y=4\sin(t)$ for $0\leq t\leq 2\pi$ and note the direction/orientation of the graph.

t	x	y
0	4	0
$\frac{\pi}{4}$	$2\sqrt{2}$	$2\sqrt{2}$
$\frac{\pi}{2}$	0	4
$\frac{3\pi}{4}$	$-2\sqrt{2}$	$2\sqrt{2}$
π	-4	0



ex. Use the calculator to graph the curve described by $x=2\cos(t)$ and $y=-2\cos(t)$ for $0 \le t \le 2\pi$ and note the direction/orientation of the graph.

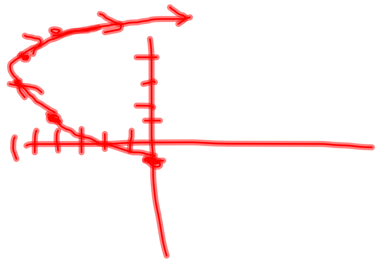
Solving one parametric equation for t and substituting into the other is called <u>eliminating the parameter</u> and is the way to change to rectangular form.

ex. Use the calculator to graph the curve described by $x=t^2-5t$ and $y=2t-1$, for t in [0,6], note the direction/orientation of the graph, and rewrite in rectangular form.

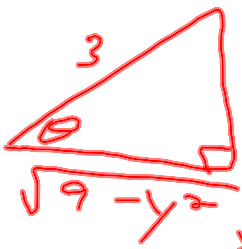
$$y+1=2t$$

$$\frac{y+1}{2}=t$$

$$x=\left(\frac{y+1}{2}\right)^2-5\left(\frac{y+1}{2}\right)$$



ex. Eliminate the parameter in x=5cos(t) and y=3sin(t) to write the equation of the curve in rectangular form.



$$\frac{y}{3} = \sin t$$

$$\arcsin\left(\frac{y}{3}\right) = t$$

$$x = 5\cos\left(\arcsin\left(\frac{y}{3}\right)\right)$$

$$x = 5\left(\frac{\sqrt{9-y^2}}{3}\right)$$

$$x = \frac{5\sqrt{9-y^2}}{3}$$

$$3x = 5\sqrt{9-y^2}$$

$$9x^2 = 25(9-y^2)$$

$$9x^2 = 225 - 25y^2$$

$$9x^2 + 25y^2 = 225$$

ex. Express $y=4x^2-1$ in parametric form 3 different ways:

a. Let $x=t$. $\rightarrow$ $x(t)=t$, $y(t)=4t^2-1$

b. Let $x=t+1$, $y=4(t+1)^2-1$

c. Let $m=dy/dx=8x$.

$m=8x$

$x=\frac{m}{8}$ $y=4\left(\frac{m}{8}\right)^2-1$

Defn. A curve C represented by x=f(t) and y=g(t) on an interval I is called smooth if f ' and g' are continuous on I and not simultaneously 0, except possibly at the endpoints of I.

ex. Determine whether x=2(t-sint) and y=2(1-cost) define a smooth curve.

$\frac{dx}{dt} = 2(1-\cos t)$ ← which are cont's $\forall t$.

$\frac{dy}{dt} = 2(\sin t)$

$2\sin t = 0$

$\sin t = 0$

$t = \ldots, 0, \pi, 2\pi, \ldots$

$t = k\pi$, k is an integer

$\frac{dx}{dt} = 0$ when $t = \ldots, 0, 2\pi, 4\pi, \ldots$

The curve is not smooth at $2k\pi$

so it's smooth on $(0, 2\pi)$

$(2\pi, 4\pi)$

$\vdots$

$(2k\pi, (2k+2)\pi)$

$2(k+1)\pi$