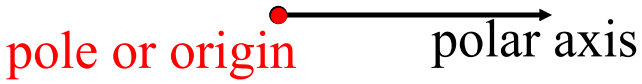
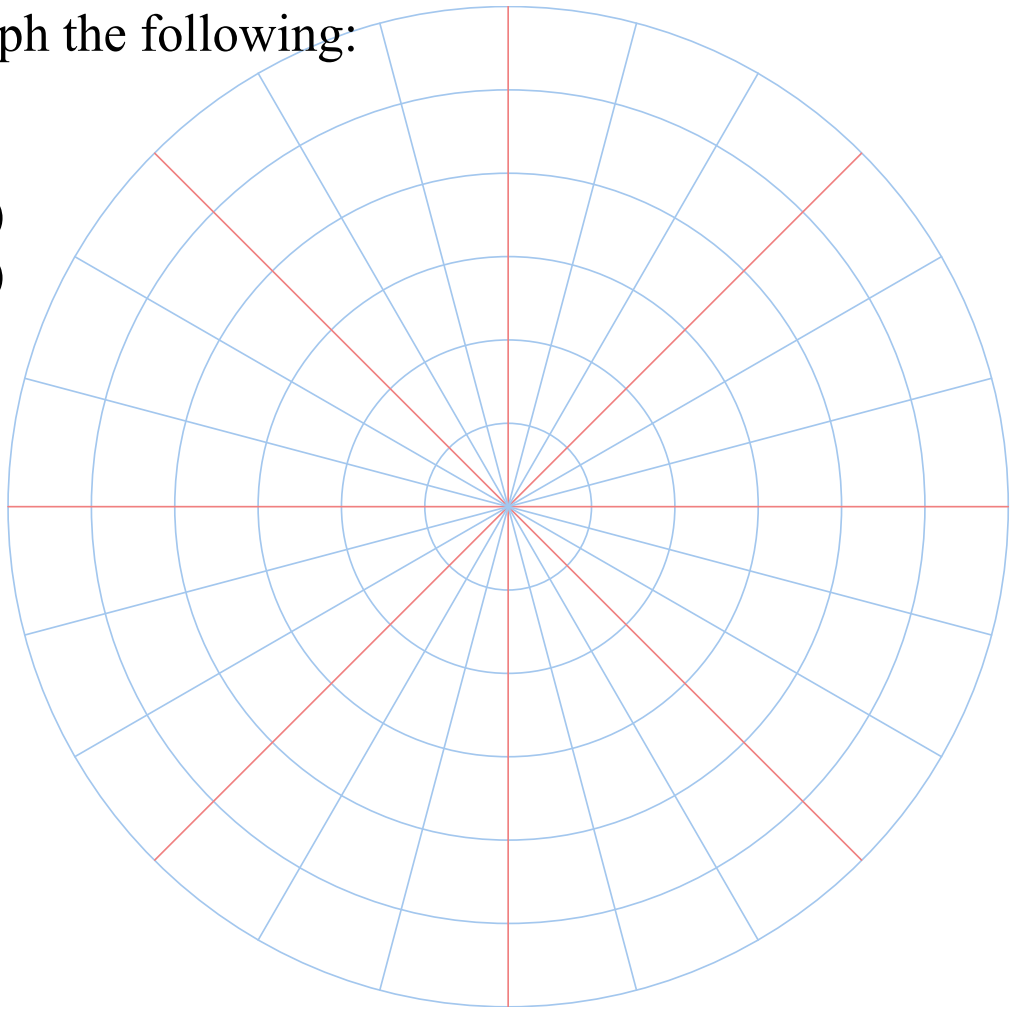
10.4 Polar coordinates and polar graphs

The polar coordinate system

polar coordinates: (r, θ)
where r=distance from the pole and θ=directed angle counterclockwise from the polar axis to ray $\overrightarrow{OP}$.



ex. Graph the following:
$(1,\pi/6)$
$(2,\pi)$
$(-3,\pi/4)$
$(2,-\pi/2)$



Thm. (r,θ) and (x,y) are related by:

$x=r\cos\theta$
$y=r\sin\theta$ } These are the trig formulas solved for x and y.

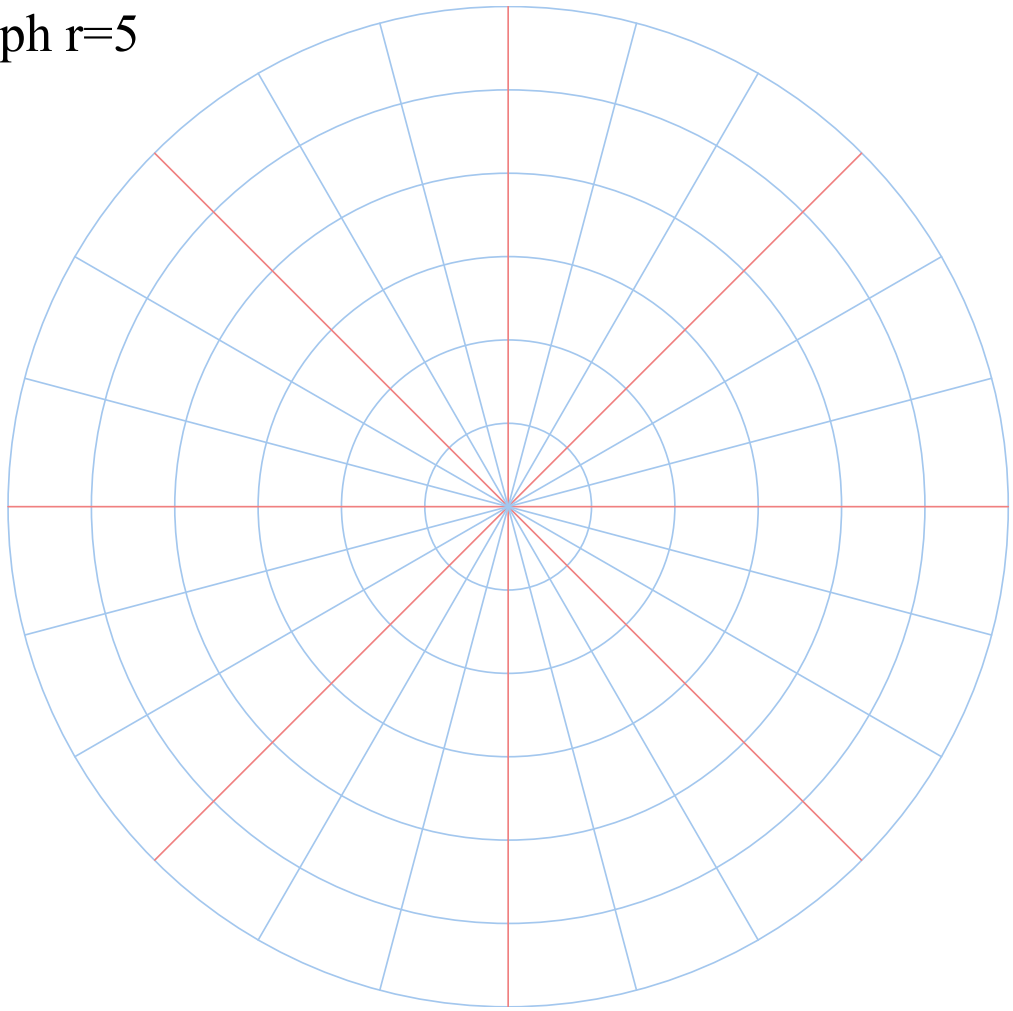
$\tan\theta=\frac{y}{x}$

$x^2+y^2=r^2$

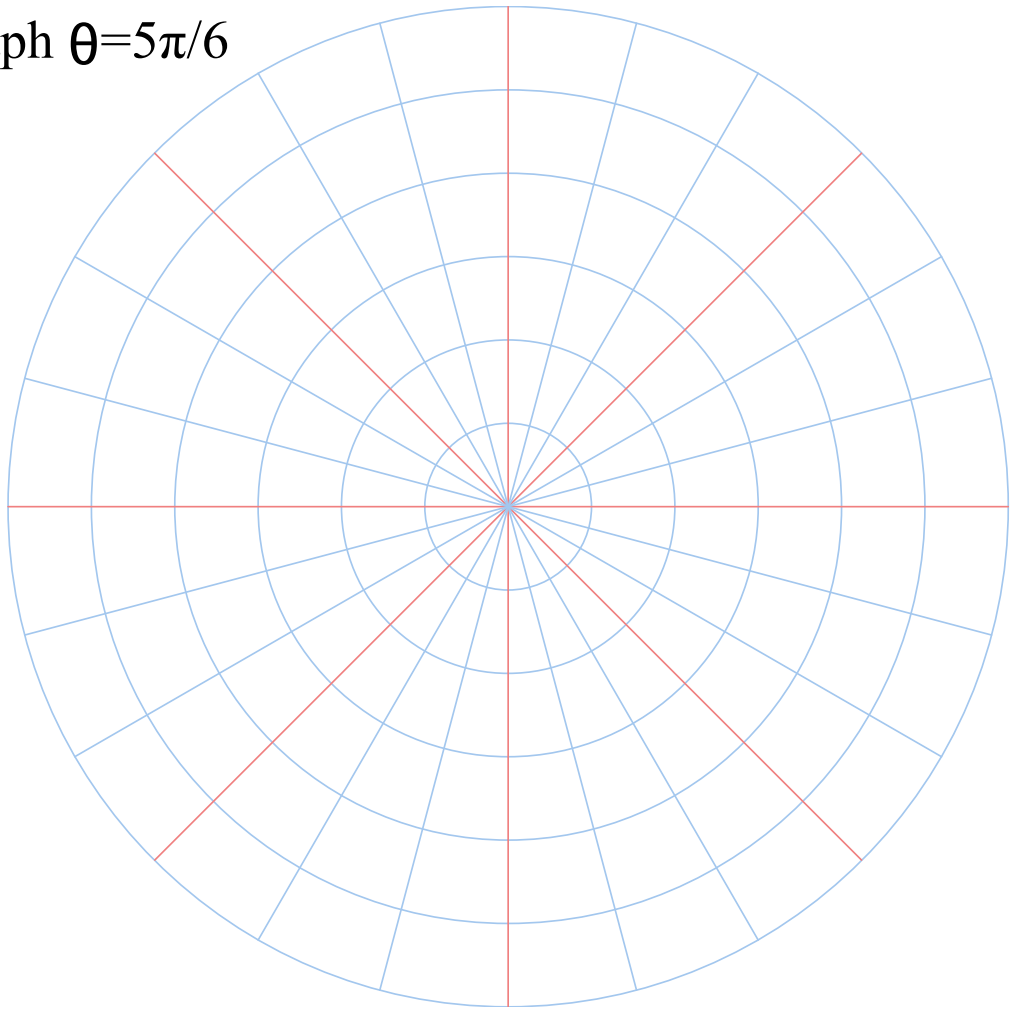
ex. Convert to polar: $(-\sqrt{3}, -1)$

ex. Convert to rectangular: $(2, 3\pi/4)$

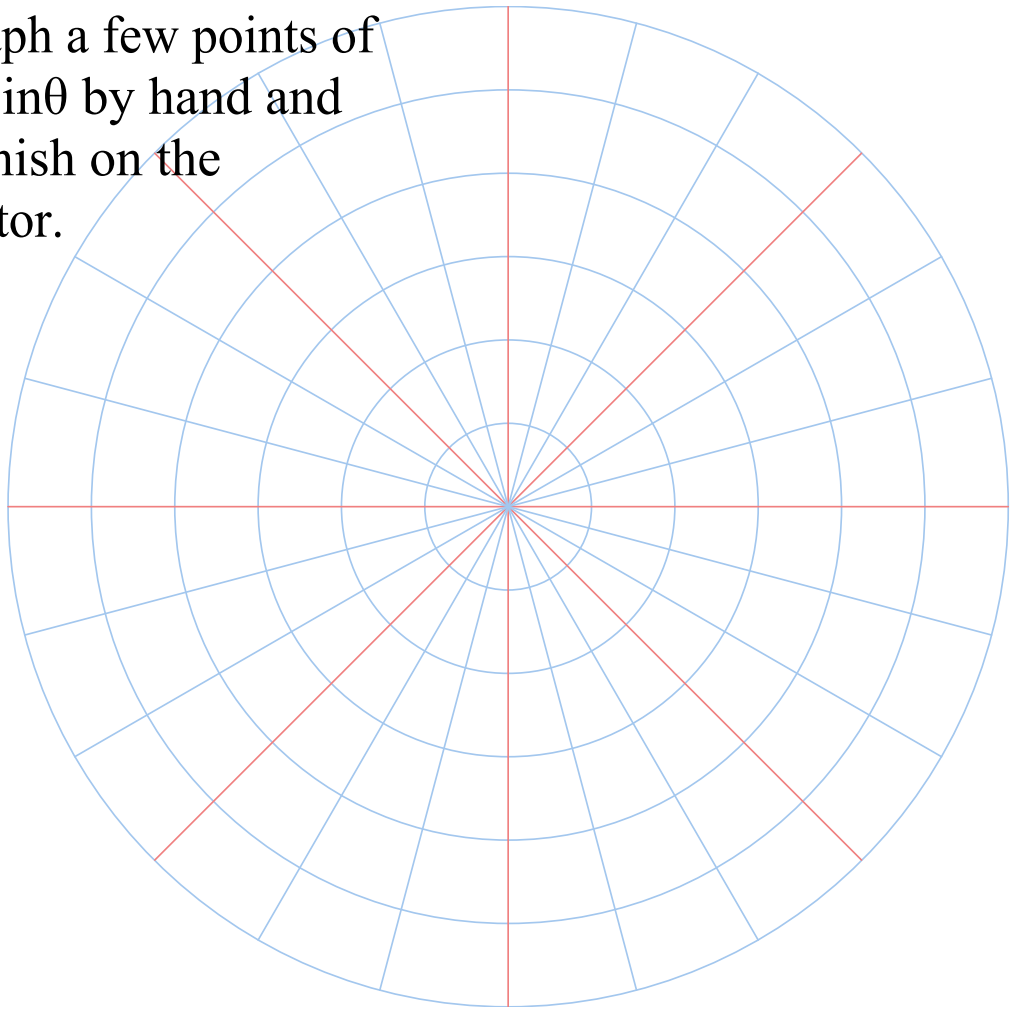
ex. Graph r=5



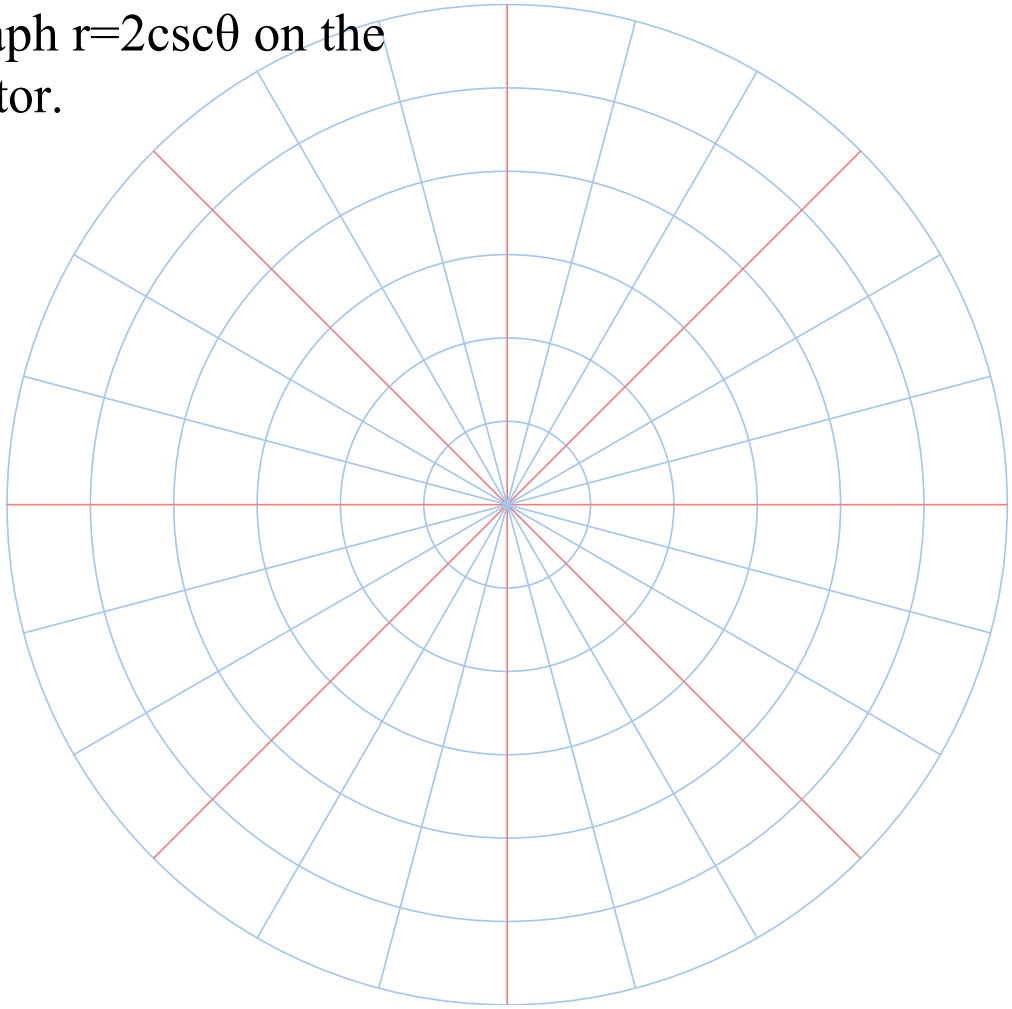
ex. Graph $\theta=5\pi/6$



ex. Graph a few points of $r=1+2\sin\theta$ by hand and then finish on the calculator.



ex. Graph $r=2\csc\theta$ on the calculator.



Thm. If f is a differentiable function of θ, then the slope of the tangent line to the graph of r=f(θ) at the point (r, θ) is

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d}{d\theta}[r\sin\theta]}{\frac{d}{d\theta}[r\cos\theta]}$$

provided that $\frac{dx}{d\theta} \neq 0$ at (r,θ)

The graphing calculator can find this.

ex. Use the calculator to find the slope of the tangent line at the point where $\theta=\pi/6$ for $r=2\sin3\theta$ and then find the equation of this tangent line.

Thm.

Given $\theta=a$
$r=0$ when $\theta=a$
$dr/d\theta \neq 0$ when $\theta=a$

The line $\theta=a$ is tangent to $r=f(\theta)$ at the pole.

ex. Verify that $\theta=0$ (or $\theta=\pi$) is tangent to $r=4\sin\theta$ at the pole.