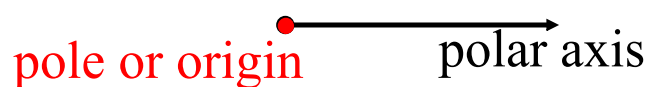


10.4 Polar coordinates and polar graphs

The polar coordinate system

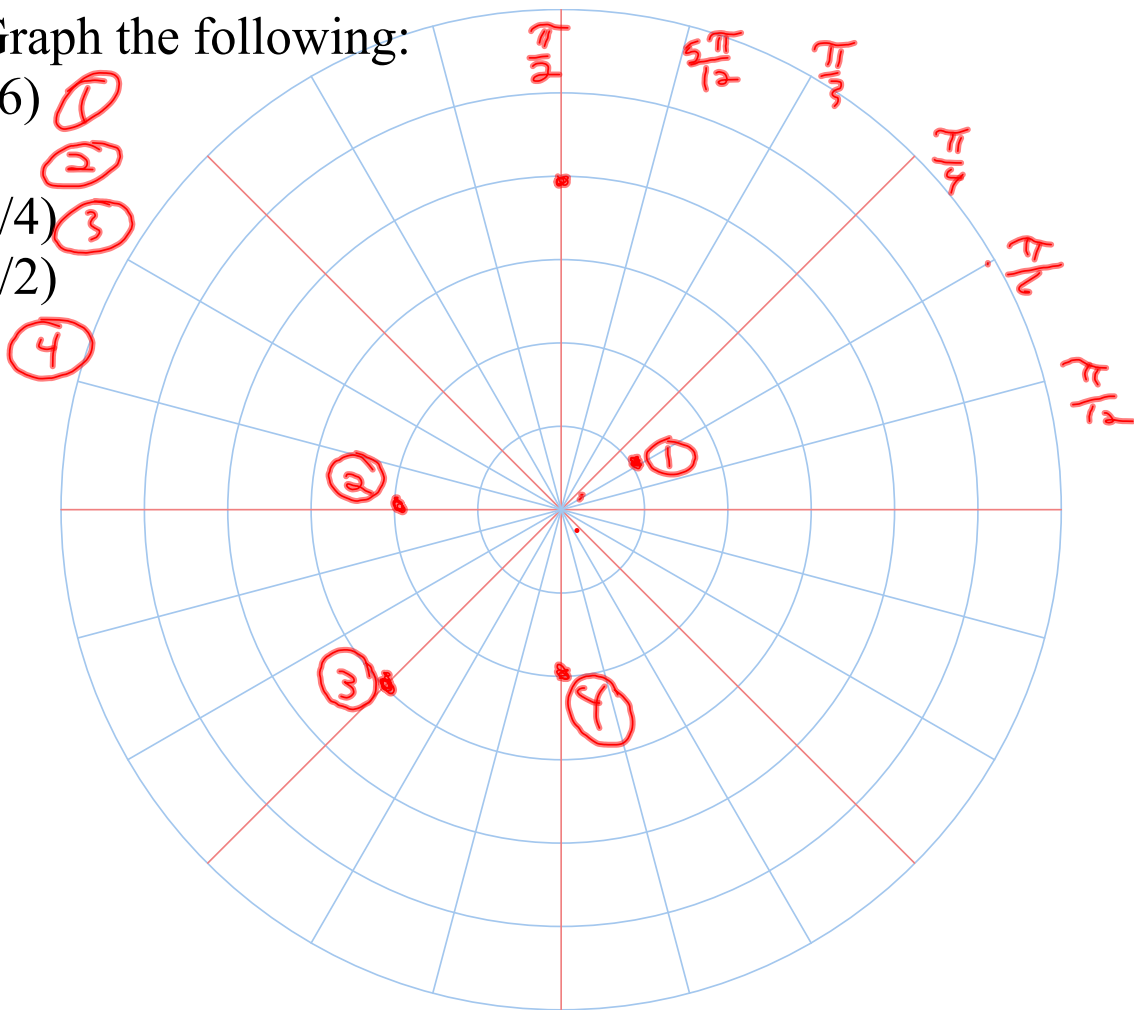
polar coordinates: (r, θ)

where r =distance from the pole and θ =directed angle counterclockwise from the polar axis to ray \overrightarrow{OP} .



ex. Graph the following:

- $(1, \pi/6)$ ①
- $(2, \pi)$ ②
- $(-3, \pi/4)$ ③
- $(2, -\pi/2)$ ④



Thm. (r, θ) and (x, y) are related by:

$x = r \cos \theta$
 $y = r \sin \theta$ } These are the trig formulas solved for x and y .

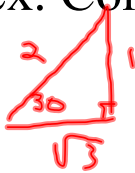
$$\tan \theta = \frac{y}{x}$$

$$x^2 + y^2 = r^2$$

$$\cos \theta = \frac{x}{r} \quad \sin \theta = \frac{y}{r}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{r} \cdot \frac{r}{x} = \frac{y}{x}$$

ex. Convert to polar: $(-\sqrt{3}, -1)$



$$\tan \theta = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6}$$

$$1^2 + (\sqrt{3})^2 = r^2$$

$$1 + 3 = r^2$$

$$2 = r^2$$

$$r = \sqrt{2}$$

$$\left(\sqrt{2}, \frac{7\pi}{6} \right)$$

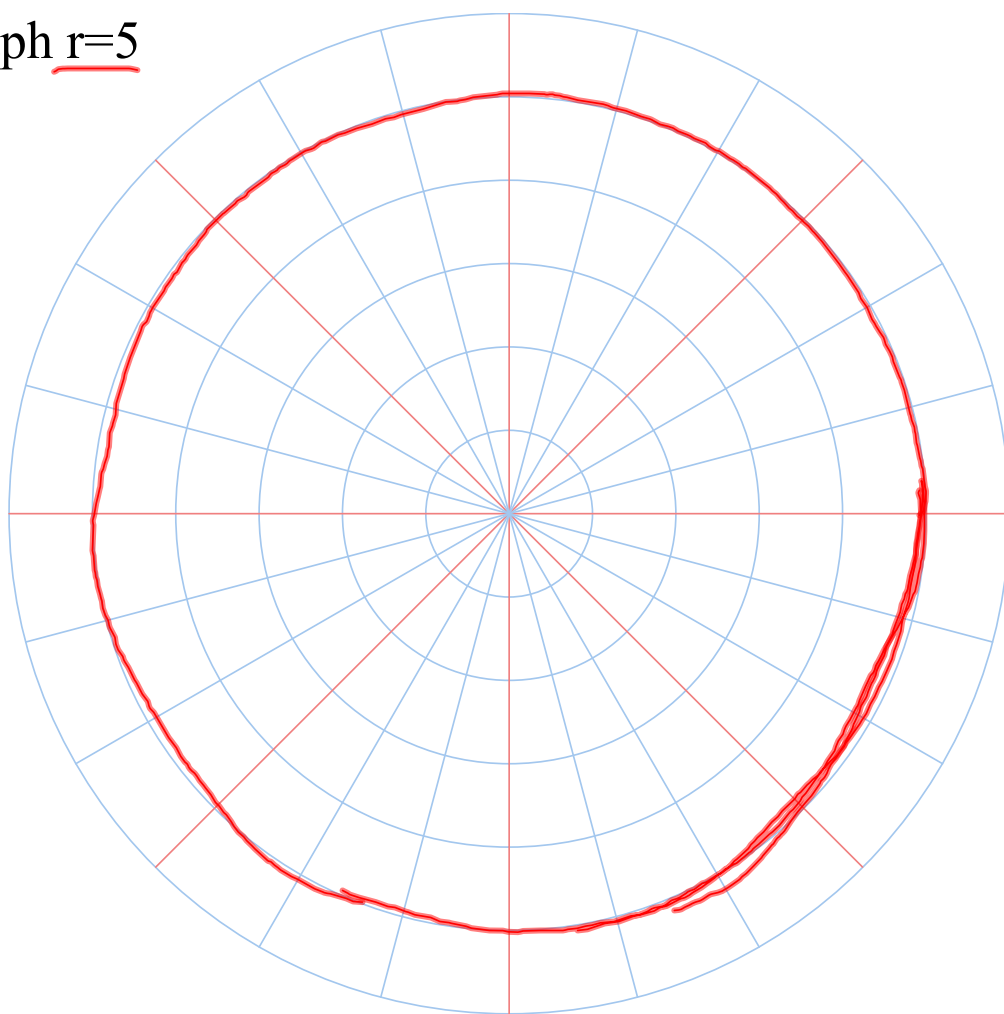
ex. Convert to rectangular: $(2, 3\pi/4)$

$$x = r \cos \theta = 2 \cos \frac{3\pi}{4} = 2 \left(-\frac{\sqrt{2}}{2} \right) = -\sqrt{2}$$

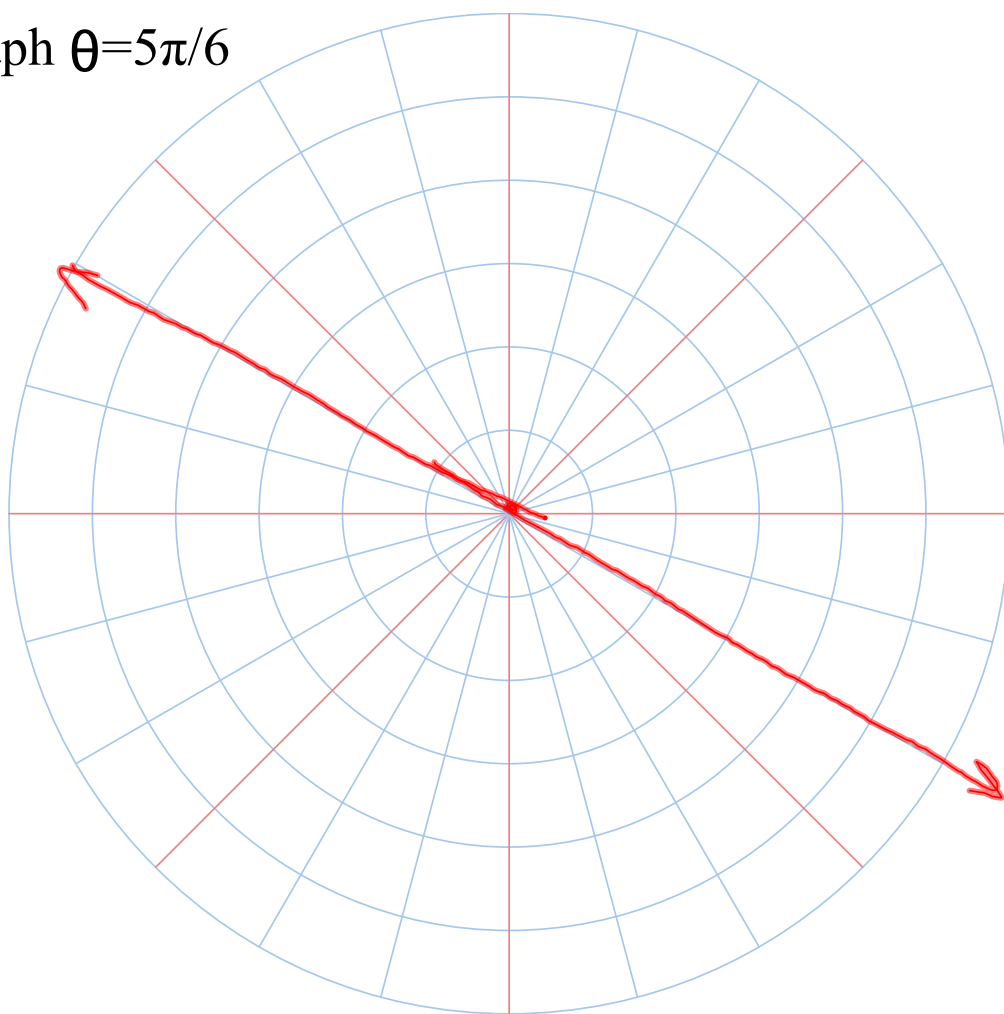
$$y = r \sin \theta = 2 \sin \frac{3\pi}{4} = 2 \left(\frac{\sqrt{2}}{2} \right) = \sqrt{2}$$

$$(-\sqrt{2}, \sqrt{2})$$

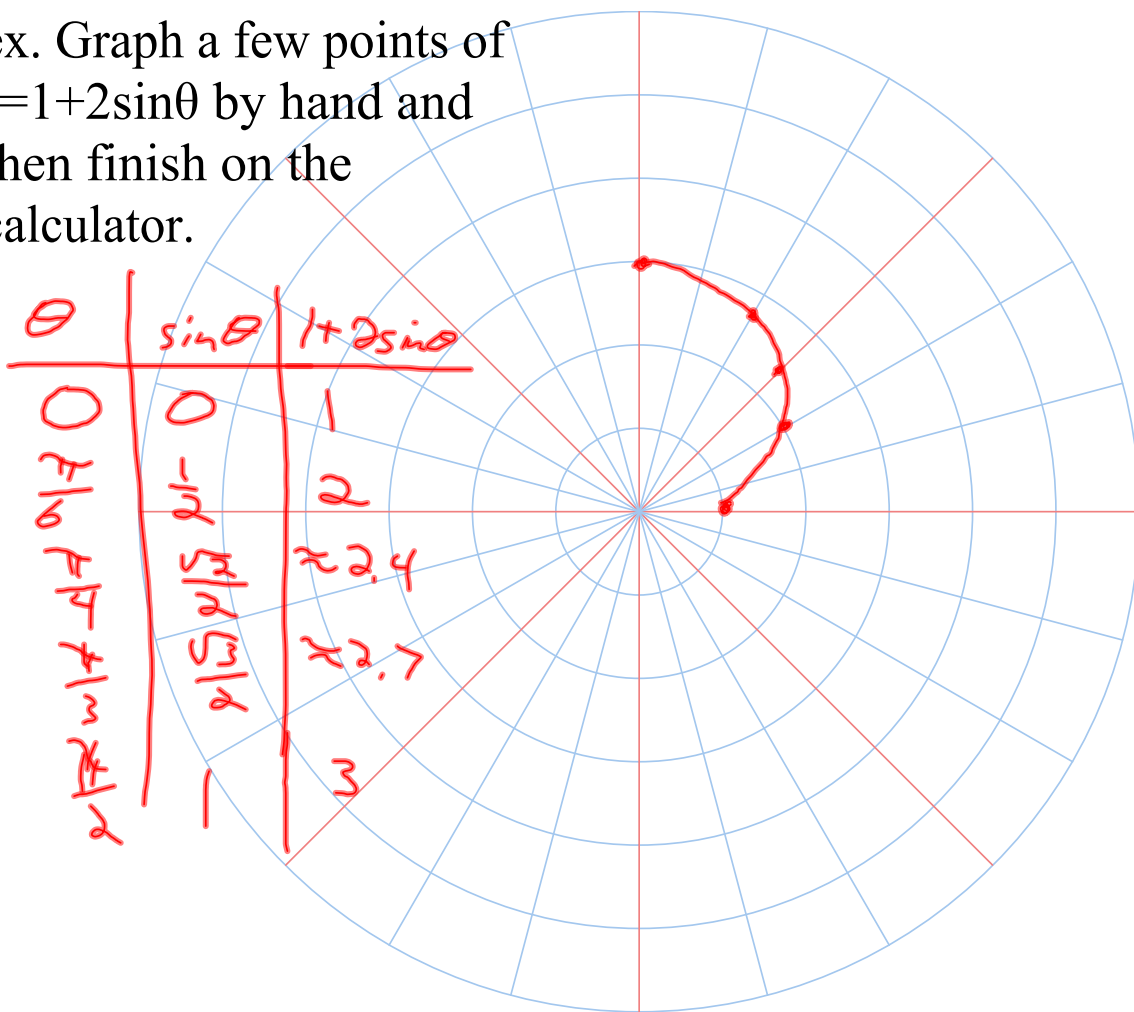
ex. Graph $r=5$



ex. Graph $\theta = 5\pi/6$

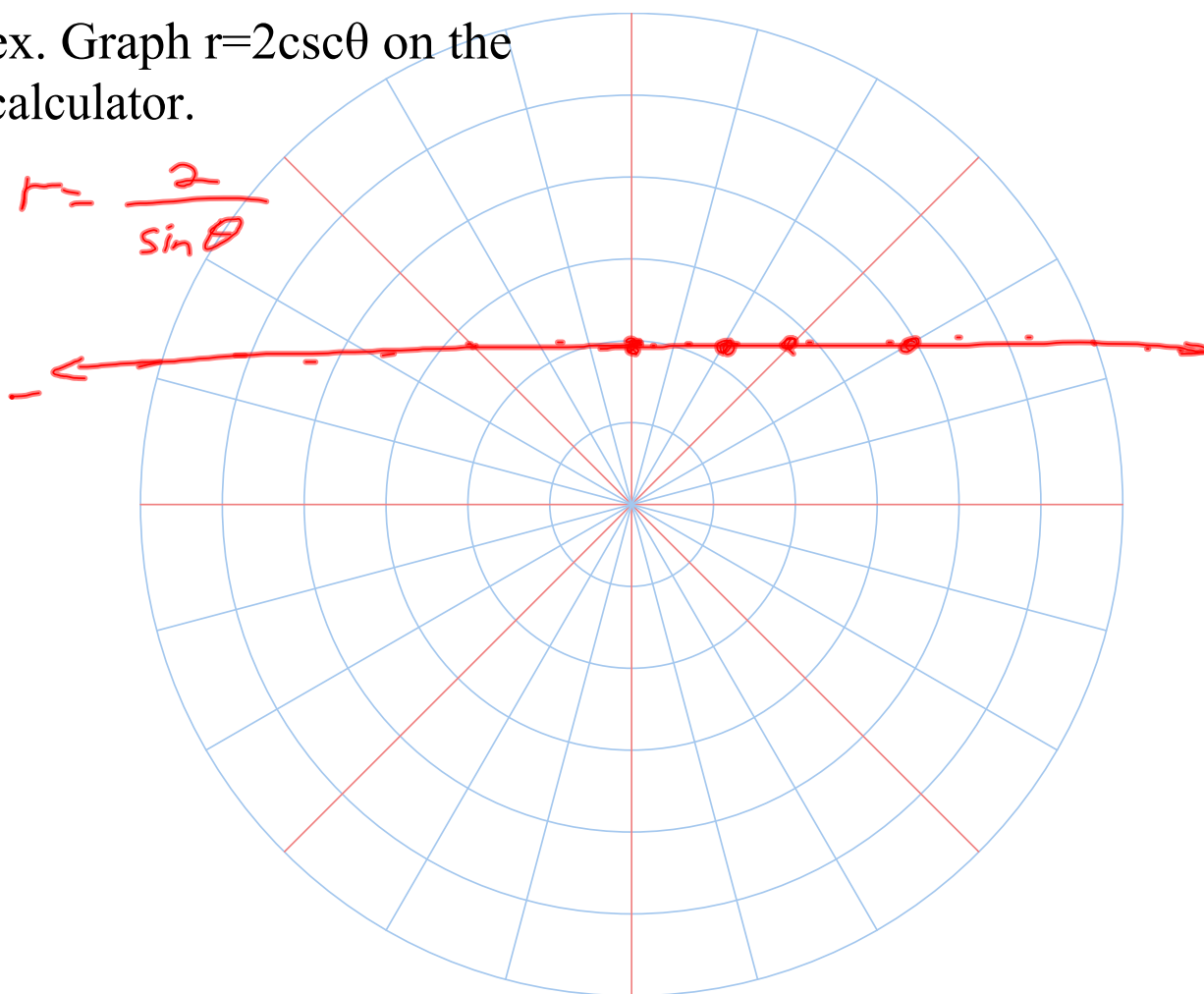


ex. Graph a few points of $r=1+2\sin\theta$ by hand and then finish on the calculator.



ex. Graph $r=2\csc\theta$ on the calculator.

$$r = \frac{2}{\sin\theta}$$



Thm. If f is a differentiable function of θ , then the slope of the tangent line to the graph of $r=f(\theta)$ at the point (r, θ) is

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d}{d\theta}[r\sin\theta]}{\frac{d}{d\theta}[r\cos\theta]}$$

provided that $\frac{dx}{d\theta} \neq 0$ at (r, θ)

The graphing calculator can find this.

ex. Use the calculator to find the slope of the tangent line at the point where $\theta = \pi/6$ for $r = 2\sin 3\theta$ and then find the equation of this tangent line.

$$(x, y) = (\sqrt{3}, 1)$$

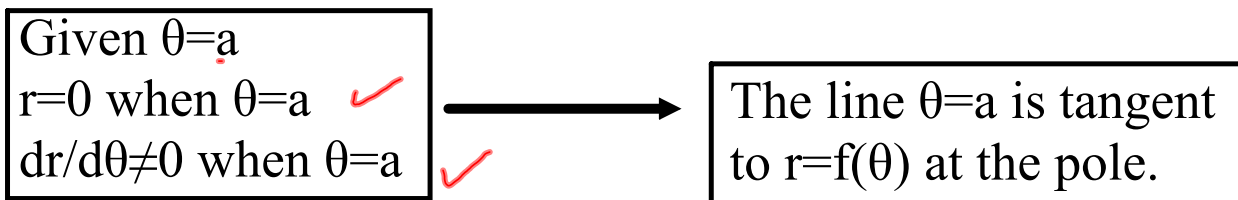
$$(1.732, 1)$$

$$\text{slope} = \frac{dy}{dx} = -\sqrt{3} \approx -1.732$$

$$y - 1 = -\sqrt{3}(x - \sqrt{3})$$

$$\frac{y - 1}{x - \sqrt{3}} = -\sqrt{3}$$

Thm.



ex. Verify that $\theta=0$ (or $\theta=\pi$) is tangent to $r=4\sin\theta$ at the pole.

$$r = 4 \sin 0 = 4 \cdot 0 = 0$$

$$\frac{dr}{d\theta} = 4 \cos \theta = 4 \cos 0 = 4(1) \neq 0$$