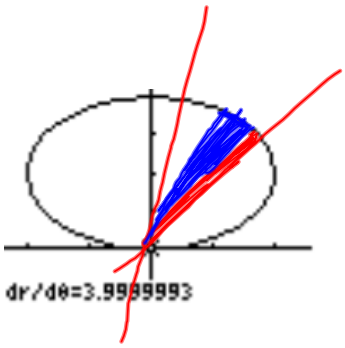
10.5 Area and Arc Length in Polar Coordinates

Recall from trig: Area of a sector of a circle is $A=\frac{1}{2}\theta r^2$ for θ in radians.

Thm. If f is continuous and nonnegative on [a,b], then the area of the region bounded by r=f(θ) between the radial lines θ=a and θ=b is given by

$$A=\frac{1}{2}\int_a^b [f(\theta)]^2\,d\theta$$

$$\text{or } \frac{1}{2}\int_a^b r^2\,d\theta$$



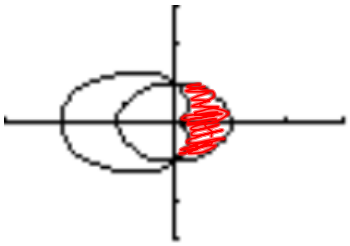
ex. Set up an integral for the area bounded by the cardioid $r=2+2\cos\theta$ and then use the calculator to evaluate this integral.

ex. Set up an integral to find the area inside the smaller loop of the limaçon $r=2\cos\theta+1$ and use the calculator to evaluate this integral.

ex. Find the area inside the circle r=1 and outside the cardioid r=1-cosθ using the formula

$$A=\frac{1}{2}\int_a^b (r_O^2-r_i^2)\,d\theta$$

and the calculator.



Though in the book, arc length and surface area for polar equations are NOT on the AP Calculus BC exam.