10.5 Area and Arc Length in Polar Coordinates

Recall from trig: Area of a sector of a circle is $A=\frac{1}{2}\theta r^2$ for θ in radians.

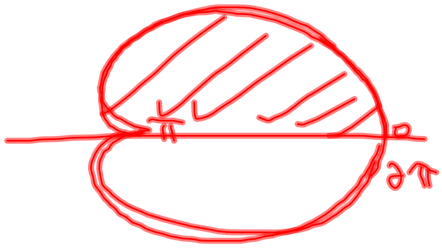
Thm. If f is continuous and nonnegative on [a,b], then the area of the region bounded by $r=f(\theta)$ between the radial lines $\theta=a$ and $\theta=b$ is given by

$$A=\frac{1}{2}\int_a^b[f(\theta)]^2\,d\theta$$

$$\text{or } \frac{1}{2}\int_a^b r^2\,d\theta$$

ex. Set up an integral for the area bounded by the cardioid r=2+2cosθ and then use the calculator to evaluate this integral.

$$\frac{1}{2}\int_0^{2\pi}(2+2\cos\theta)^2\,d\theta$$



$$= 18.850$$

$$= \cancel{\frac{1}{2}}\cdot\cancel{2}\int_0^{\pi}(2+2\cos\theta)^2\,d\theta$$

ex. Set up an integral to find the area inside the smaller loop of the limaçon r=2cosθ +1 and use the calculator to evaluate this integral.

$$\frac{1}{2}\int_{2.094}^{4.189} (2\cos\theta+1)^2\, d\theta = .544$$

$4.189 \leftarrow \frac{4\pi}{3}$

$2.094 \leftarrow \frac{2\pi}{3}$

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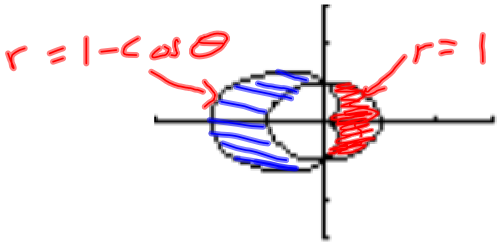
.5fnInt((1+2cos(
θ))²,θ,A,B)
        .5435164422
.5fnInt((1+2cos(
θ))²,θ,2π/3,4π/3
)
        .5435164422
```

ex. Find the area inside the circle r=1 and outside the cardioid r=1-cosθ using the formula

$$A=\frac{1}{2}\int_a^b (r_o^2-r_i^2)\, d\theta$$

and the calculator.



The curves intersect at $\frac{\pi}{2}$ & $\frac{3\pi}{2}$

$$\text{Area} = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(1^2 - (1-\cos\theta)^2\right) d\theta$$

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.5fnInt((1-(1-co
s(θ))²),θ,-π/2,π
/2)
            1.214601837
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Though in the book, arc length and surface area for polar equations are NOT on the AP Calculus BC exam.