10.6 Polar equations of conics and Kepler's Laws

Thm. Classifying conics The locus of points in the plane whose distance from a fixed point (focus) has a constant ratio to its distance to a fixed line (directrix) is a conic. The constant e is the eccentricity of the conic.

The conic is a(n)

- ellipse for $0<e<1$,
- parabola for $e=1$,
- hyperbola for $e>1$.

Thm. Polar equations of conics

$$r=\frac{ed}{1\pm e\cos\theta} \quad \text{or} \quad r=\frac{ed}{1\pm e\sin\theta}$$

is a conic, where $e>0$ is the eccentricity and $|d|$ is the distance between the focus at the pole and its corresponding directrix.

Note:
sin--> horizontal directrix
+esin$\boldsymbol{\theta}$ above the pole
-esin$\boldsymbol{\theta}$ below the pole

cos--> vertical directrix
+ecos$\boldsymbol{\theta}$ right of the pole
-ecos$\boldsymbol{\theta}$ left of the pole

ex. Sketch $r=\frac{6}{2+\cos\theta}$

ex. Sketch $r=\frac{2}{1+2\sin\theta}$

Kepler's Laws:

1. (orbits) Each planet moves in an elliptical path with sun at one focus.
2. (areas) Equal areas are swept out in equal times by the segment from the sun to the planet.
3. (periods) The square of a planet's orbital period is proportional to the cube of the length of the semimajor axis of its elliptical orbit.

ex. Halley's comet has e about 0.97. find a polar equation for its orbit and what is its aphelion (farthest distance from the sun)? Assume that 2a=36.18 AU.

ex. Halley's comet has a period of about 76 years. How long does it take to move from the position given by $\boldsymbol{\theta}=0$ to $\boldsymbol{\theta}=\pi$?