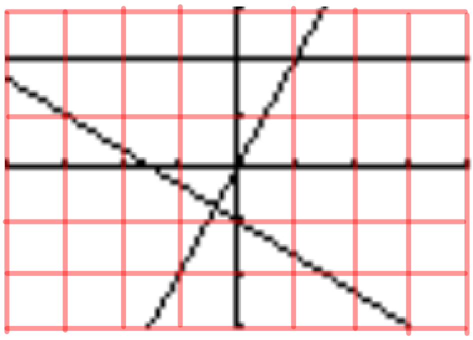
Do now as a warm-up:

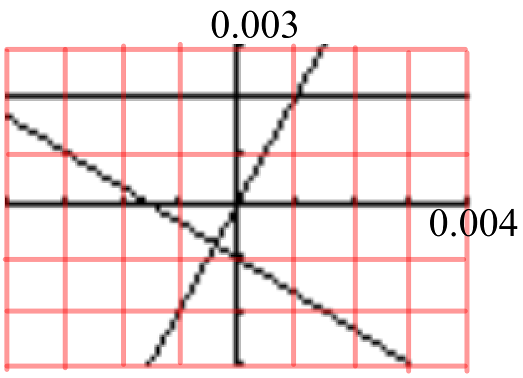
1. Enter any function of your choosing in Y_1.
2. Press ZOOM and then press 6.
3. Press 2nd, press TRACE, then press 6.
4. Enter any number from -10 to 10 and press ENTER.
5. Repeat steps 1-4 until you think you can answer this question:

What does this sequence of steps do?

2.1 The Derivative and the Tangent Line Problem



What is the equation of each function graphed?



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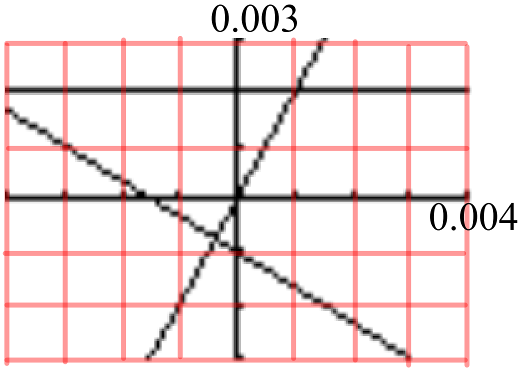

The 3 functions graphed here are:

$$f(x) = x^3 + 0.002$$

$$g(x) = -\frac{2}{3}x - 0.001$$

$$h(x) = \sin(2x)$$

Which is which?

How do you know?

Graph these functions in a standard window.

$$f(x) = |x-2| + 1$$

$$g(x) = 2\sqrt[3]{x-2} + 1$$

$$h(x) = (x-2)^2 + 1$$

$$m(x) = (\sin(x-2))^2 + 1$$

$$p(x) = 4(x-2)^{\frac{2}{3}} + 1$$

$$q(x) = \frac{x-1}{x-2}$$

$$r(x) = \tan(x - \frac{\pi}{2} - 2) + 1$$

$$v(x) = x^3 + x^2 - 2x - 7$$

Then trace to the area near where x=2 and zoom in on that region.

2.1 The Derivative and the Tangent Line Problem

Applets of secant line and tangent line

http://www.ies.co.jp/math/java/calc/limsec/limsec.html

http://www.slu.edu/classes/maymk/Applets/SecantTangent.html

http://www.calvin.edu/~rpruim/courses/m161/F01/java/SecantTangent.shtml

Applet of derivative of curve at a point

http://www.univie.ac.at/future.media/moe/galerie/diff1/diff1.html

Classic slope formula (picnic table):

Slope formula for a secant line:

$$m_{\sec} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(c+\Delta x) - f(c)}{(c+\Delta x) - c} = \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

$$m_{\tan} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \to x_1} \frac{y_2 - y_1}{x_2 - x_1} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{(c+\Delta x) - c} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

http://www.calculusapplets.com/avevel.html

Defn. Tangent line with slope m

If f is defined on an open interval containing c and the limit

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x} = m$$

exists, then the line passing through the point (c,f(c)) with slope m is the tangent line to the graph of f at (c, f(c)).

This also defines the derivative of f at x=c:

$$m_{\tan} = f'(c) = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

Ex. Use the definition to find the slope where x=2 for the function

$f(x) = \sqrt{3x-4}$

Ex. For the function $f(x)=x^2+1$
find the slope of the tangent line at:

a. (0,1)

b. (-1,2)

If the limit (the slope of the tangent line) = $\pm\infty$, then the tangent line is a vertical line.

Defn. The derivative of f at x:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

as long as the limit exists.

Defn. The process of finding derivatives is called differentiation.

Defn. A function is differentiable on (a,b) if the derivative exists $\forall x \in (a,b)$.

3 ways a function is NOT differentiable at a point:
1. slope goes to $\pm\infty$ (vertical tangent line) 2. a cusp or kink (f is cont's but f ' is not cont's) 3. f is discont's

Notation used for derivatives:

$f'(x)$ $\quad \frac{dy}{dx}$ $\quad y'$ $\quad D_x$ $\quad \frac{d}{dx}\left[f(x)\right]$

Thm.

f is continuous at c

$\lim_{x \to c^-} f'(x) = \lim_{x \to c^+} f'(x)$

→ f is differentiable at c

Thm.

f is differentiable at c → f is continuous at c http://www.ima.umn.edu/~arnold/calculus/secants/secants3/secants-g.html

http://www.ima.umn.edu/~arnold/calculus/secants/secants4/secants-g.html

http://www.calculusapplets.com/makecontdiff.html

Ex. Use the definition to find f '(x) for $f(x) = \frac{1}{x}$

Now try it in reverse!

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^2-2(x+h)+3-x^2+2x-3}{h}$$

What is f(x)?

$$f'(x)=\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$$

What is f(x)?

$$f'(a)=\lim_{h\to 0}\frac{\tan\left(\frac{\pi}{3}+h\right)-\sqrt{3}}{h}$$

What is f(x) and what is a?

Thm. If f is differentiable at x=c, then f is continuous at x=c.

Remember, for any theorem, the contrapositive is also true!

Contrapositive: If f is **<u>not</u>** continuous at x=c, then f is **<u>not</u>** differentiable at x=c.

The inverse and the converse are not necessarily true!

Converse: If f is continuous at x=c, then f is differentiable at x=c. **<u>False!</u>**

Inverse: If f is not differentiable at x=c, then f is not continuous at x=c. **<u>False!</u>**

Alternate form of the derivative:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

useful for the derivative at a point

Ex. If $f(x)=|x-2|$, use the alternate form to find $f'(x)$ at $x=2$.

Note:

f is differentiable at c —— Not necessarily ——→ f ' is continuous at c

ex. $$f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$$

This is cont's and diff'ble, but f ' is not cont's.