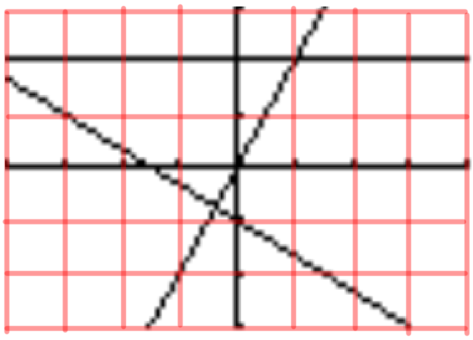
Do now as a warm-up:

1. Enter any function of your choosing in Y_1.
2. Press ZOOM and then press 6.
3. Press 2nd, press TRACE, then press 6.
4. Enter any number from -10 to 10 and press ENTER.
5. Repeat steps 1-4 until you think you can answer this question:

What does this sequence of steps do?

2.1 The Derivative and the Tangent Line Problem

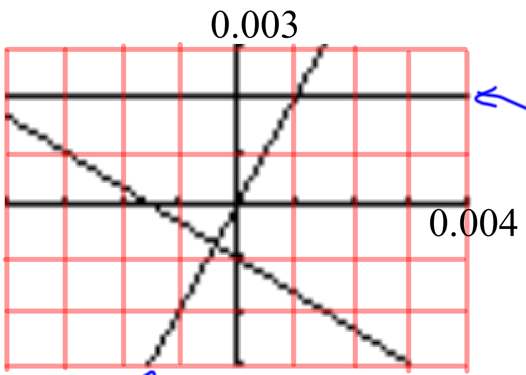


What is the equation of each function graphed?

$y = 2$

$y = -\frac{2}{3}x - 1$

$y = 2x$

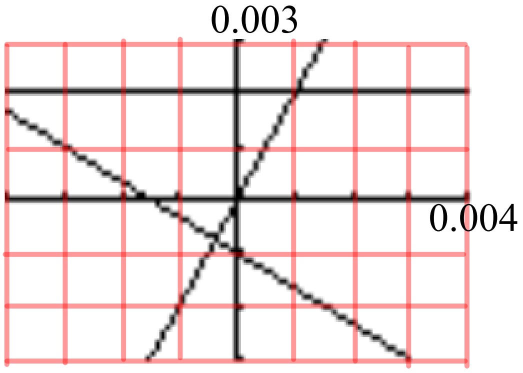


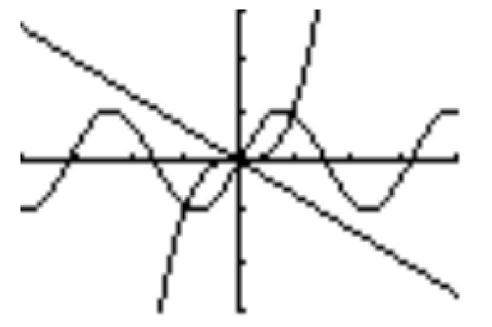
What is the equation of each function graphed?

$y = .002$

$y = 2x$

$y = -\frac{2}{3}x - .001$





The 3 functions graphed here are:

$$f(x) = x^3 + 0.002$$

$$g(x) = -\frac{2}{3}x - 0.001$$

$$h(x) = \sin(2x)$$

Which is which?

How do you know?

Graph these functions in a standard window.

Then trace to the area near where x=2 and zoom in on that region.

not diff'ble at x=2 $f(x)=|x-2|+1$

g'(2) is undef'd so not diff'ble $g(x)=2\sqrt[3]{x-2}+1$

h'(2) = 0 $h(x)=(x-2)^2+1$

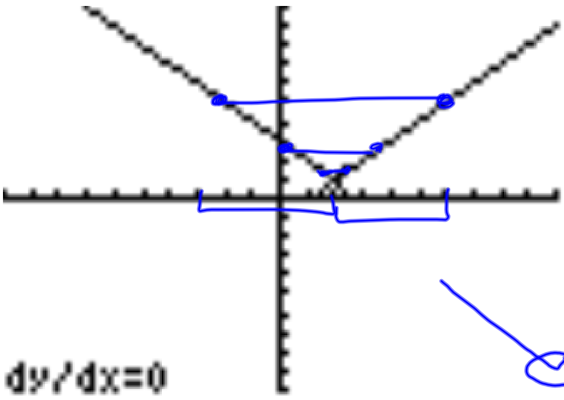
m'(2) = 0 $m(x)=(\sin(x-2))^2+1$

p is not diff'ble at x=2 $p(x)=4(x-2)^{\frac{2}{3}}+1$

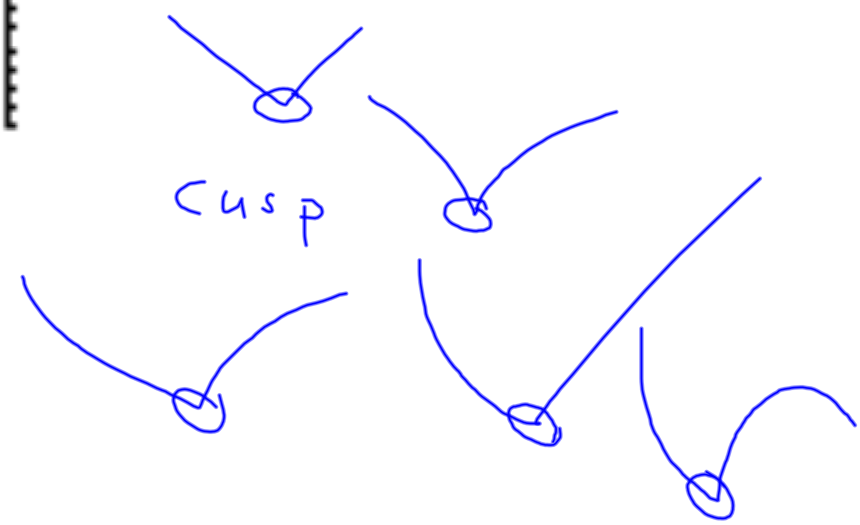
q not cont's at x=2 so q is not diff'ble at x=2 $q(x)=\dfrac{x-1}{x-2}$

r is like q $r(x)=\tan(x-\frac{\pi}{2}-2)+1$

should get v'(2) = 14 → $v(x)=x^3+x^2-2x-7$



wrong!



2.1 The Derivative and the Tangent Line Problem

Applets of secant line and tangent line

http://www.ies.co.jp/math/java/calc/limsec/limsec.html

http://www.slu.edu/classes/maymk/Applets/SecantTangent.html

http://www.calvin.edu/~rpruim/courses/m161/F01/java/SecantTangent.shtml

Applet of derivative of curve at a point

http://www.univie.ac.at/future.media/moe/galerie/diff1/diff1.html

Classic slope formula (picnic table):

$(c, f(c))$

Slope formula for a secant line:

$$m_{\sec} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(c+\Delta x) - f(c)}{(c+\Delta x) - c} = \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

$(c + \Delta x, f(c + \Delta x))$

$$m_{\tan} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \to x_1} \frac{y_2 - y_1}{x_2 - x_1} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{(c+\Delta x) - c} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

http://www.calculusapplets.com/avevel.html

Defn. Tangent line with slope m

If f is defined on an open interval containing c and the limit

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x} = m$$

exists, then the line passing through the point (c,f(c)) with slope m is the tangent line to the graph of f at (c, f(c)).

This also defines the derivative of f at x=c:

$$m_{\tan} = f'(c) = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

Ex. Use the definition to find the slope where x=2 for the function

$$f(x)=\sqrt{3x-4}$$

$$f'(2)=\lim_{\Delta x\to 0}\frac{\sqrt{3(2+\Delta x)-4}-\sqrt{2}}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{\sqrt{2+3\Delta x}-\sqrt{2}}{\Delta x}\cdot\frac{\sqrt{2+3\Delta x}+\sqrt{2}}{\sqrt{2+3\Delta x}+\sqrt{2}}$$

$$=\lim_{\Delta x\to 0}\frac{(\cancel{2}+3\cancel{\Delta x})-\cancel{2}}{\cancel{\Delta x}(\sqrt{2+3\Delta x}+\sqrt{2})}=\frac{3}{2\sqrt{2}}$$

Ex. For the function $f(x)=x^2+1$
find the slope of the tangent line at:

a. (0,1)

$c \quad f(c)$

$$\text{slope} = \lim_{\Delta x \to 0} \frac{(0+\Delta x)^2 \cancel{+1} \cancel{-1}}{\Delta x} = \lim_{\Delta x \to 0} \frac{(\Delta x)^{\cancel{2}}}{\cancel{\Delta x}} = 0$$

b. (-1,2)

$c \quad f(c)$

$$\lim_{h \to 0} \frac{(-1+h)^2+1-2}{h} = \lim_{h \to 0} \frac{\cancel{1}-2h+h^2 \cancel{-1}}{h}$$

$$= \lim_{h \to 0} -2+h = -2$$

If the limit (the slope of the tangent line) = $\pm\infty$, then the tangent line is a vertical line.

Defn. The <u>derivative</u> of f at x:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

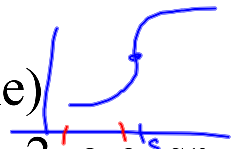
as long as the limit exists.

Defn. The process of finding derivatives is called <u>differentiation</u>.

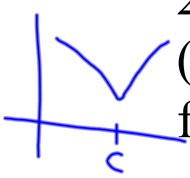
Defn. A function is <u>differentiable on (a,b)</u> if the derivative exists $\forall x \in (a,b)$.

3 ways a function is NOT differentiable at a point:

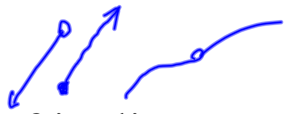
1. slope goes to $\pm\infty$ (vertical tangent line)

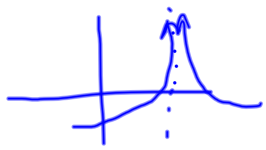


2. a cusp or kink (f is cont's but f ' is not cont's)



3. f is discont's





Notation used for derivatives:

$f'(x)$ $\quad \frac{dy}{dx}$ $\quad y'$ $\quad D_x$ $\quad \frac{d}{dx}\left[f(x)\right]$

Thm.

$\boxed{\begin{array}{l}\text{f is continuous at c} \\ \lim_{x \to c^-} f'(x) = \lim_{x \to c^+} f'(x)\end{array}} \longrightarrow \boxed{\text{f is differentiable at c}}$

Thm.

$\boxed{\text{f is differentiable at c}} \longrightarrow \boxed{\text{f is continuous at c}}$

not con't's ⟶ not diff'ble

http://www.ima.umn.edu/~arnold/calculus/secants/secants3/secants-g.html

http://www.ima.umn.edu/~arnold/calculus/secants/secants4/secants-g.html

http://www.calculusapplets.com/makecontdiff.html

Ex. Use the definition to find f '(x) for $f(x) = \dfrac{1}{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{x}{x(x+\Delta x)} - \frac{x+\Delta x}{x(x+\Delta x)}}{\frac{\Delta x}{1}}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x} - \cancel{x} - \overset{-1}{\cancel{\Delta x}}}{x(x+\Delta x)} \cdot \frac{1}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{x(x+\Delta x)} = \frac{-1}{x^2}$$

Now try it in reverse!

$-(x^2-2x+3)$

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^2-2(x+h)+3-x^2+2x-3}{h}$$

What is f(x)?

$= x^2-2x+3$

$2x-2$

$$f'(x)=\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$$

What is f(x)?

$$f'(a)=\lim_{h\to 0}\frac{\tan\left(\frac{\pi}{3}+h\right)-\sqrt{3}}{h}$$

$\tan\frac{\pi}{3}$

What is f(x) and what is a?

$\tan x$ $\frac{\pi}{3}$

Thm. If f is differentiable at x=c, then f is continuous at x=c.

Remember, for any theorem, the contrapositive is also true!

Contrapositive: If f is **not** continuous at x=c, then f is **not** differentiable at x=c.

The inverse and the converse are not necessarily true!

Converse: If f is continuous at x=c, then f is differentiable at x=c. **False!**

Inverse: If f is not differentiable at x=c, then f is not continuous at x=c. **False!**

Alternate form of the derivative:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

useful for the derivative at a point

$$x^2 + x + 1$$

$$f'(1) = \lim_{x \to 1} \frac{x^3 - 1}{x - 1} = 3$$

$f(x) = x^3$

$f'(x) = 3x^2$

Ex. If f(x)=|x-2|, use the alternate form to find f '(x) at x=2.

$$f'(2) = \lim_{x\to 2} \frac{|x-2| - 0}{x-2} \quad \text{DNE}$$

$$\lim_{x\to 2^-} \frac{|x-2|}{x-2} = -1$$

$$\lim_{x\to 2^+} \frac{|x-2|}{x-2} = 1$$

Note:

$\boxed{\text{f is differentiable at c}} \xrightarrow[\text{necessarily}]{\text{Not}} \boxed{\text{f ' is continuous at c}}$

ex. $f(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

This is cont's and diff'ble, at x=0 but f ' is not cont's.