Do now as a warm-up:

Let's see what you remember about the shortcuts. Try finding these derivatives without using the limit definition.

1. $f(x) = 6$

2. $f(x) = x$

3. $f(x) = x^5$

4. $f(x) = 7x^5$

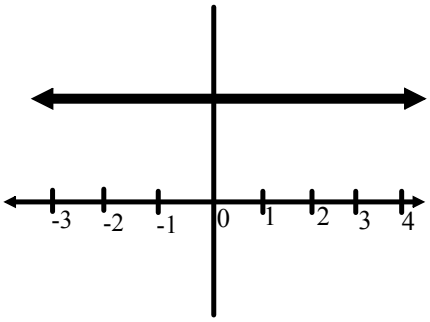
5. $f(x) = 2x^3 + 4x^2 - 7x - 5$

2.2 Basic Differentiation Rules and Rates of Change

Thm. The Constant Rule

$$\frac{d}{dx}[c]=0$$

Pf. $\lim_{\Delta x\to 0}\frac{c-c}{\Delta x}=\lim_{\Delta x\to 0}\frac{0}{\Delta x}=0$



ex. Find f '(x) if f(x) = 3

Thm. The Power Rule

$$\frac{d}{dx}x^n = nx^{n-1}$$

ex. Find f '(x) if f(x) = x^3

Thm. The Constant Multiple Rule

$$\frac{d}{dx}\left[cf(x)\right]=cf'(x)$$

ex. $\frac{d}{dx}\left[3x^7\right]=$

Thm. The Sum or Difference Rule

$$\frac{d}{dx}\left[f(x)\pm g(x)\right]=f'(x)\pm g'(x)$$

ex. Find f '(x) if f(x) = $4x^3 - 2x^2 + x - 5$

ex. Find the x coordinates of all points on

$$y = x^3 + 3x + 2$$

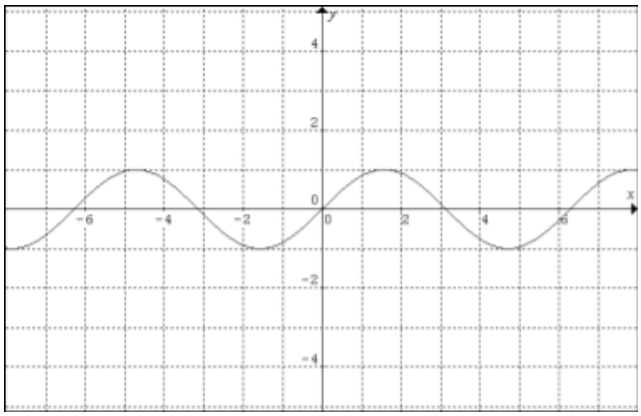
where a tangent line would be horizontal.

ex. Find f '(x) if f(x)= $\frac{2}{x^4}$

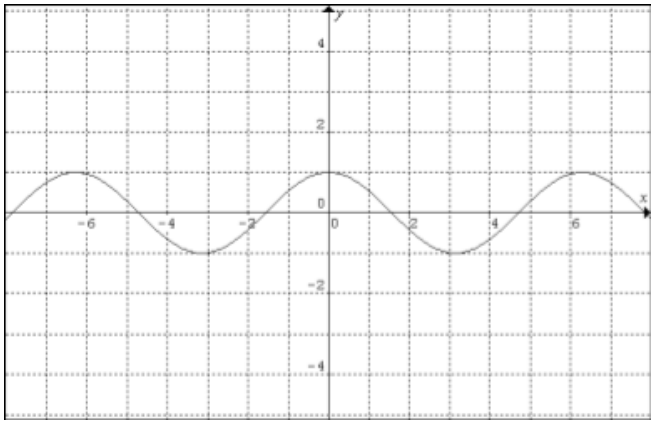
ex. Find f '(x) if f(x)= $\sqrt[4]{x^3}$

Thm. Derivatives of sin and cos

$\frac{d}{dx} \sin x$



$\frac{d}{dx} \cos x$



The derivative is a function, too!

Relate slope of tangent line to a function to the graph of derivative of that function

http://clem.mscd.edu/%7Etalmanl/HTML/MovingSlopeTriangle.html

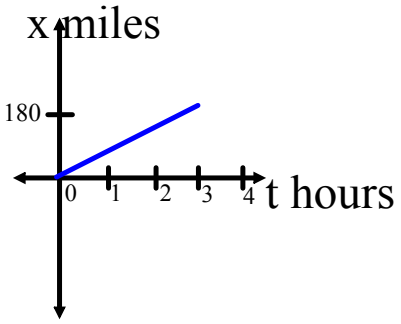
Your calculator can graph the derivative!

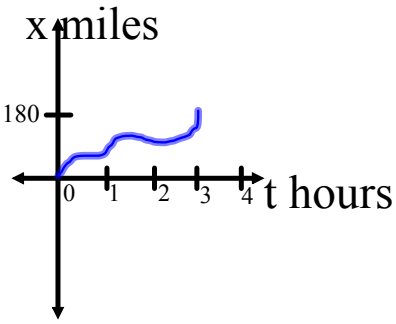
Def. x(t) is the position function. It gives the position of an object relative to the origin.

velocity = distance / time

average velocity = $\frac{\Delta d}{\Delta t} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = slope = m$

=the rate of change of x per unit t





For either trip shown, the average velocity is $\frac{180-0}{3-0} = \frac{180}{3} = 60mph$

$$\text{instantaneous velocity} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = x'(t)$$

physics	calc	English	metaphor	metric units	rate of change
x	x	position	mother	m	——
v	x'	velocity	kid	m/s	$\Delta x/\Delta t$
a	x''	acceleration	grandkid	m/s^2	$\Delta v/\Delta t$

Is that speed or velocity?

Speed is always positive.

Speed is a magnitude.

Velocity could be positive or negative.

Velocity is a vector.

Speed = |Velocity|

Speed = Velocity ONLY when the position is increasing.
That means the velocity will be positive.

For vertical motion = $x(t)=\frac{1}{2}gt^2+v_0t+x_0$

g is the constant acceleration due to gravity
v_o is the original or initial velocity
x_o is the original or initial position
$g= -9.8\ m/s^2$ or $g= -32\ ft/sec^2$

Notice that the acceleration for gravity is a constant:

$x(t) = -4.9t^2+v_0t+x_0$
$x'(t) = v(t) = -9.8t+v_o$
$x''(t) = -9.8 = a(t)$

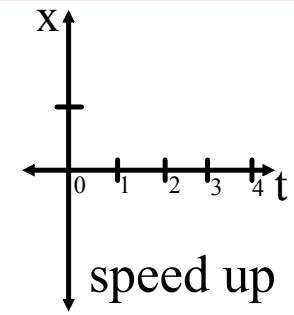
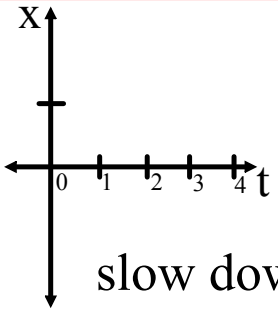
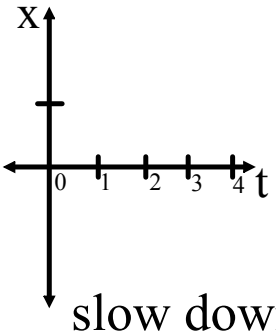
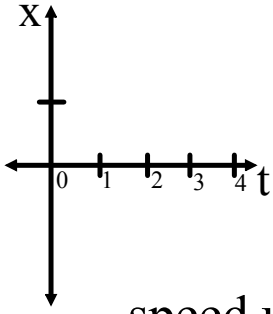
Linear motion

		at rest, stopped	moving forward up, or right		moving backward down, or left		turning around, changing direction
position	x(t)						
velocity	v(t)	0	+		-		0 and change sign
speed	|v(t)|	0	faster	slower	faster	slower	0
acceleration	a(t)	unknown	+	-	-	+	unknown

v(t) and a(t) same signs ⟶ speeding up

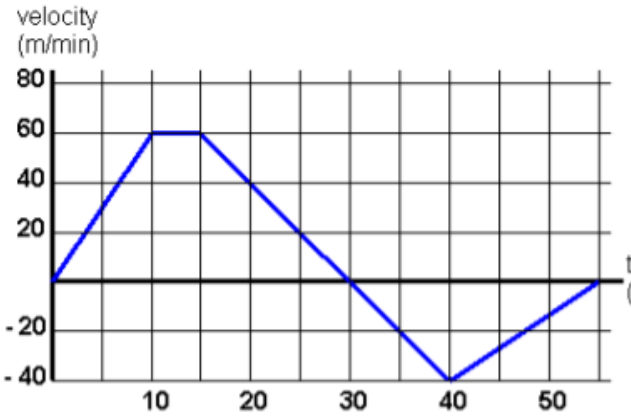
v(t) and a(t) opposite signs ⟶ slowing down

Sketch a position-time graph for each situation:

	v(t)>0	v(t)<0
a(t)>0	 speed up	 slow down
a(t)<0	 slow down	 speed up

ex. Describe the direction, velocity, and speed for a particle that moves according to the equation

$$x(t) = -t^2 + 3t$$



ex. When is the particle...
at rest?

going fastest?

moving to the right?

moving to the left?

When is the particle's acceleration...
positive?

negative?

0?

ex. A particle moves along a horizontal line so that its position at any time $t \geq 0$ is given by $x(t) = t^3 - t^2 - t + 3$

a. Find the instantaneous velocity of the particle at t=1.
b. Find the average velocity of the particle on the interval [0,3].
c. For what value(s) of t on the interval [0,3] is the particle's instantaneous velocity the same as its average velocity on the interval [0,3]?