Do now as a warm-up:

Let's see what you remember about the shortcuts. Try finding these derivatives without using the limit definition.

1. $f(x) = 6$ $\quad f'(x) = 0$

2. $f(x) = x$ $\quad f'(x) = 1$

3. $f(x) = x^5$ $\quad f'(x) = 5x^4$

4. $f(x) = 7x^5$ $\quad f'(x) = 35x^4$

5. $f(x) = 2x^3 + 4x^2 - 7x - 5$

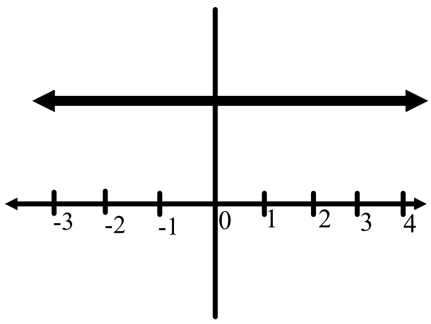
$f'(x) = 6x^2 + 8x - 7$

2.2 Basic Differentiation Rules and Rates of Change

Thm. The Constant Rule

$$\frac{d}{dx}[c]=0$$

Pf. $\lim\limits_{\Delta x \to 0} \frac{c-c}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{0}{\Delta x} = 0$



ex. Find f '(x) if f(x) = 3

$f'(x) = 0$

Thm. The Power Rule

$$\frac{d}{dx}x^n = nx^{n-1}$$

ex. Find f '(x) if f(x) = x^3

$f'(x) = 3x^2$

Thm. The Constant Multiple Rule

$$\frac{d}{dx}\left[cf(x)\right]=cf'(x)$$

ex. $\frac{d}{dx}\left[3x^7\right]=21x^6$

Thm. The Sum or Difference Rule

$$\frac{d}{dx}\left[f(x)\pm g(x)\right]=f'(x)\pm g'(x)$$

ex. Find f '(x) if f(x) = $4x^3 - 2x^2 + x - 5$

$f'(x) = 12x^2 - 4x + 1$

ex. Find the x coordinates of all points on

$$y = x^3 + 3x + 2$$

where a tangent line would be horizontal.

$$y' = 3x^2 + 3$$

$$0 = 3x^2 + 3$$

$$0 = x^2 + 1$$

$$x = \pm i$$

$\nexists$ any horizontal tangent lines

ex. Find f '(x) if f(x)= $\frac{2}{x^4}$ $= 2x^{-4}$

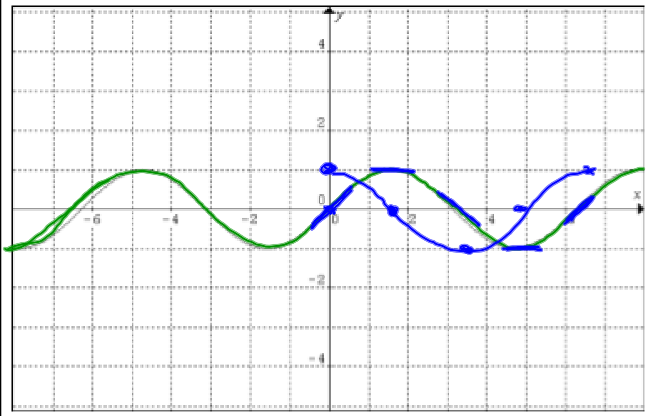
$f'(x) = -8x^{-5} = \frac{-8}{x^5}$

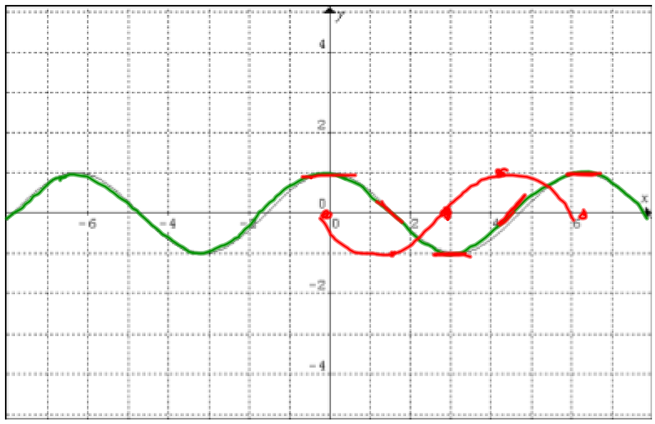
ex. Find f '(x) if f(x)= $\sqrt[4]{x^3}$ $= x^{\frac{3}{4}}$

$f'(x) = \frac{3}{4}x^{-\frac{1}{4}}$ or other equivalent

Thm. Derivatives of sin and cos

$\dfrac{d}{dx}\sin x = \cos x$ $\qquad$ $\dfrac{d}{dx}\cos x = -\sin x$





The derivative is a function, too!

Relate slope of tangent line to a function to the graph of derivative of that function

http://clem.mscd.edu/%7Etalmanl/HTML/MovingSlopeTriangle.html

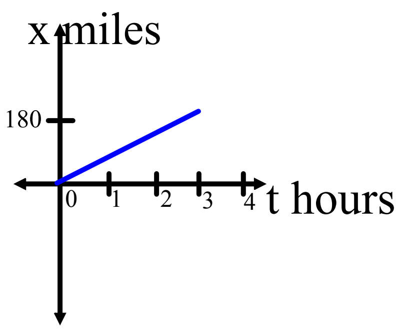
Your calculator can graph the derivative!

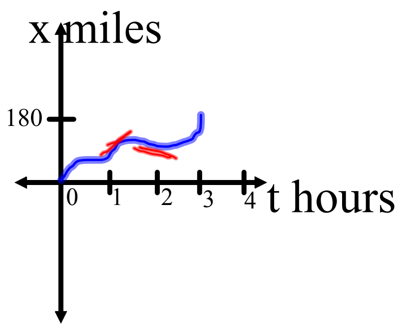
Def. x(t) is the position function. It gives the position of an object relative to the origin.

velocity = distance / time

average velocity = $\dfrac{\Delta d}{\Delta t} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_2 - x_1}{t_2 - t_1} = slope = m$

=the rate of change of x per unit t





For either trip shown, the average velocity is $\dfrac{180-0}{3-0} = \dfrac{180}{3} = 60mph$

instantaneous velocity = $\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$ = x'(t)

physics	calc	English	metaphor	metric units	rate of change
x	x	position	mother	m	—
v	x'	velocity	kid	m/s	$\Delta x/\Delta t$
a	x"	acceleration	grandkid	m/s^2	$\Delta v/\Delta t$

Is that speed or velocity?

Speed is always positive.

Velocity could be positive or negative.

Speed is a magnitude.

Velocity is a vector.

Speed = |Velocity|

Speed = Velocity ONLY when the position is increasing.
That means the velocity will be positive.

For vertical motion = $x(t)=\frac{1}{2}gt^2 + v_o t + x_o$

g is the constant acceleration due to gravity
v_o is the original or initial velocity
x_o is the original or initial position
$g= -9.8\ m/s^2$ or $g= -32\ ft/sec^2$

Notice that the acceleration for gravity is a constant:

$x(t) = -4.9t^2 + v_o t + x_o$
$x'(t) = v(t) = -9.8t + v_o$
$x''(t) = -9.8 = a(t)$

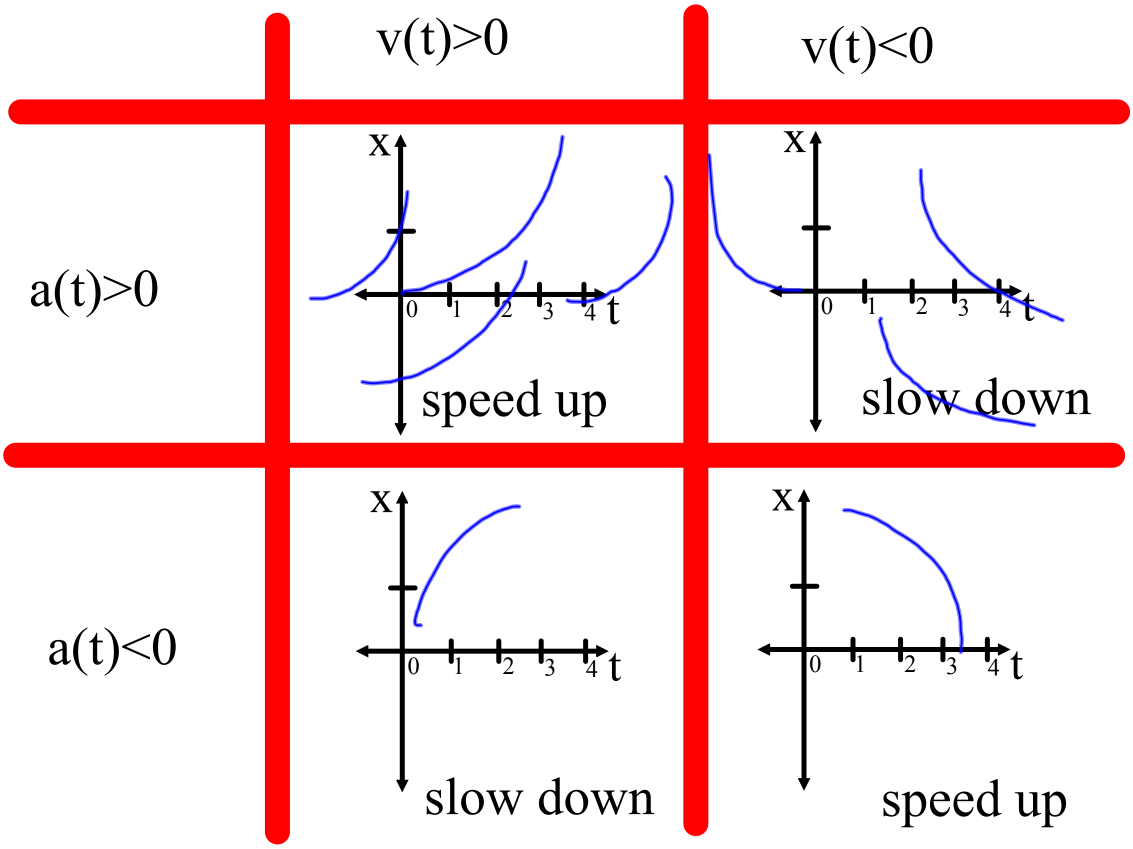
Linear motion

		at rest, stopped	moving forward up, or right		moving backward down, or left		turning around, changing direction
position	$x(t)$						
velocity	$v(t)$	0	+		-		0 and change sign
speed	$\lvert v(t)\rvert$	0	faster	slower	faster	slower	0
acceleration	$a(t)$	unknown	+	-	-	+	unknown

$v(t)$ and $a(t)$ same signs $\longrightarrow$ speeding up

$v(t)$ and $a(t)$ opposite signs $\longrightarrow$ slowing down

Sketch a position-time graph for each situation:



ex. Describe the direction, velocity, and speed for a particle that moves according to the equation

$$x(t) = -t^2 + 3t$$

$v(t) = x'(t) = -2t + 3$

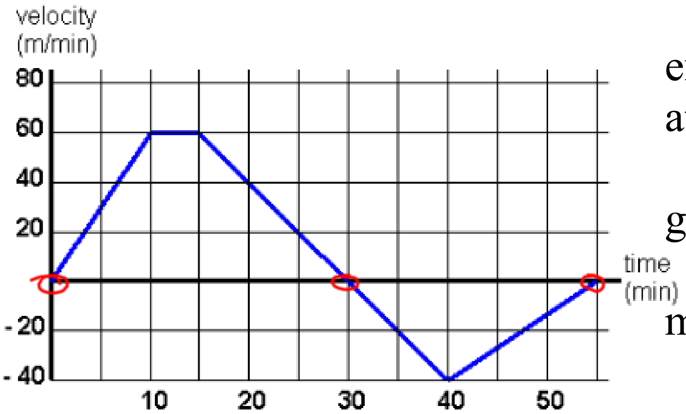
$-2t + 3 = 0$

$t = \frac{-3}{-2} = \frac{3}{2}$ At $t = \frac{3}{2}$, $v(t) = 0 =$ speed.

the particle turns at this time.

On $(-\infty, \frac{3}{2})$, $v(t) > 0$, so the particle is moving in the positive direction. The particle's speed is $v(t)$.

On $(\frac{3}{2}, \infty)$, $v(t) < 0$, so the particle is moving in the negative direction. The particle's speed is $-v(t)$.



ex. When is the particle...

at rest? at $t = 0, 30, 55$

going fastest? on $(10, 15)$ $10 \le t \le 15$

moving to the right? on $(0, 30)$

moving to the left? $(30, 55)$

When is the particle's acceleration...

positive? on $(0, 10)$ & $(40, 55)$

negative? $(15, 40)$

0? $(10, 15)$

ex. A particle moves along a horizontal line so that its position at any time t≥0 is given by $x(t) = t^3 - t^2 - t + 3$

a. Find the instantaneous velocity of the particle at t=1.
b. Find the average velocity of the particle on the interval [0,3].
c. For what value(s) of t on the interval [0,3] is the particle's instantaneous velocity the same as its average velocity on the interval [0,3]?

a. $v(t) = x'(t) = 3t^2 - 2t - 1$

$v(1) = 3 - 2 - 1 = 0$

b. $\dfrac{x(3) - x(0)}{3-0} = \dfrac{27 - 9 - 3 + 3 - 3}{3} = \dfrac{15}{3} = 5$

c. $v(t) = 5$

$3t^2 - 2t - 1 = 5$

$3t^2 - 2t - 6 = 0$

$t \approx 1.786$ $\quad t \approx -1.120$

$\in [0,3]$