Do now as a warm-up:

Do you remember how to take the derivative of functions like these?

$$f(x) = (x-2)(x+3)$$

$$f(x) = \frac{x+5}{x-2}$$

## 2.3 The Product and Quotient Rules

Thm. The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

"each one takes a turn"

$$\frac{d}{dx}[uv] = u'v + uv'$$

ex. Find y ' if $y = (x^2+2x)(x+1)$

$y' = (2x+2)(x+1) + (x^2+2x)(1)$

ex. Find f '(x) if

$f(x) = (\sin x)\sqrt{x+5}$ ← $\sqrt{x} = x^{\frac{1}{2}}$

$f'(x) = (\cos x)(\sqrt{x+5}) + (\sin x)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$

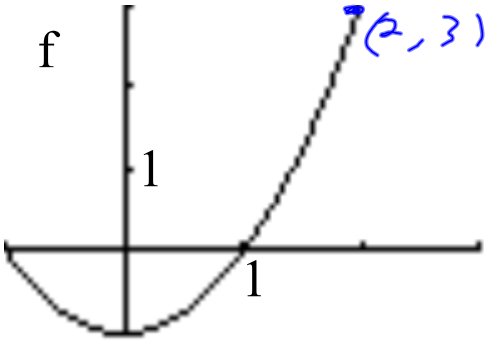
ex. Suppose h(x)=f(x)g(x).
Find h'(1) if f(1) = 2, f '(1)=-3, g(1)=5, and g'(1)=7.

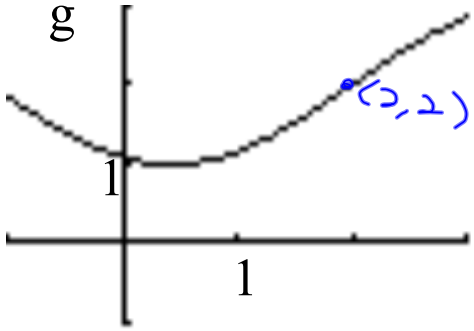
$$h'(x) = f'(x)g(x) + f(x)g'(x)$$
$$h'(1) = (-3)(5) + (2)(7)$$
$$= -15 + 14$$
$$= -1$$

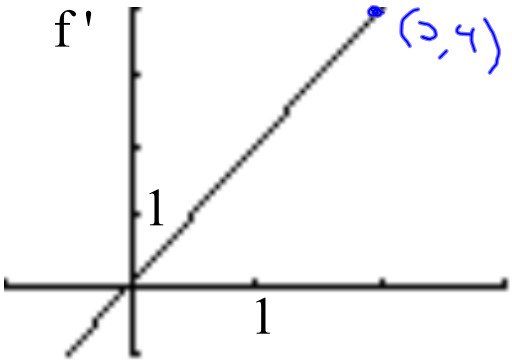
ex. Suppose h(x)=f(x)g(x).
Find h'(2).

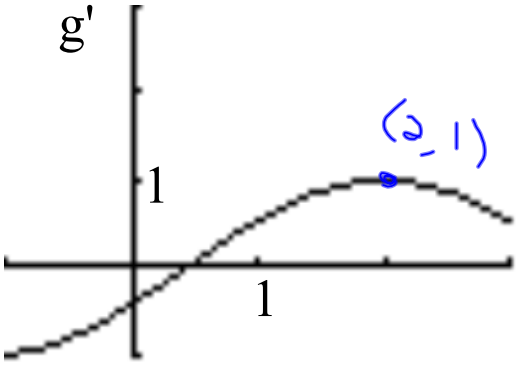
$h'(x) = f'(x)g(x) + f(x)g'(x)$

$= (4)(2) + (3)(1) = 11$



Thm. The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\frac{\text{dnum denom - num ddenom}}{\text{denom denom}} \qquad \frac{\text{low dhigh - high dlow}}{\text{low low}}$$

$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v - uv'}{v^2}$$

ex. Find the derivative of

$$f(x) = \frac{x^7}{3x-2}$$

$$f'(x) = \frac{7x^6(3x-2) - x^7(3)}{(3x-2)^2}$$

ex. Suppose h(x)=f(x)/g(x).
Find h'(1) if f(1) = 2, f '(1)=-3, g(1)=5, and g'(1)=7.

$$h'(x) = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{[g(x)]^2}$$

$$h'(1) = \frac{(-3)(5) - (2)(7)}{5^2}$$

$$= \frac{-15 - 14}{25} = \frac{-29}{25}$$

ex. $\frac{d}{dx}\tan x = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$

$= \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{(\cos x)^2}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$= \sec^2 x \quad \text{or} (\sec x)^2$

Thm. Derivatives of Trig Functions

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

ex. $\frac{d}{dx}[4\tan x - 3\sec x]$

$= 4\sec^2 x - 3\sec x \tan x$

ex. Find y ' if y= $\frac{1-\cos x}{\sin x}$ $= \frac{1}{\sin x} - \frac{\cos x}{\sin x} = \csc x - \cot x$

$y' = \frac{(+\sin x)(\sin x) - (1-\cos x)(\cos x)}{\sin^2 x}$

$= \frac{\sin^2 x - \cos x + \cos^2 x}{\sin^2 x}$

$= \frac{1 - \cos x}{\sin^2 x}$

$y' = -\csc x \cot x - (-\csc^2 x)$

$y' = \frac{-1}{\sin x} \cdot \frac{\cos x}{\sin x} + \frac{1}{\sin^2 x}$

$= \frac{-\cos x}{\sin^2 x} + \frac{1}{\sin^2 x}$

## Higher Order Derivatives

1st Derivative = $y'(x)=f'(x)=\frac{dy}{dx}=\frac{d}{dx}\left[f(x)\right]$

2nd Derivative = $y''(x)=f''(x)=\frac{d^2y}{dx^2}=\frac{d^2}{dx^2}\left[f(x)\right]$

3rd Derivative = $y'''(x)=f'''(x)=\frac{d^3y}{dx^3}=\frac{d^3}{dx^3}\left[f(x)\right]$

4th Derivative = $y^{(4)}(x)$

5th Derivative = $y^{(5)}(x)$

*etc.*

Derivative on the calculator

ex. Find the slope of the curve at x=2 for the function $y=x^2-3x+1$

2nd TRACE 6: dy/dx
Push 2
Push ENTER

MATH 8: nDeriv(
enter this: x^2-3x+1,x,2

nDeriv(expression, variable, value)