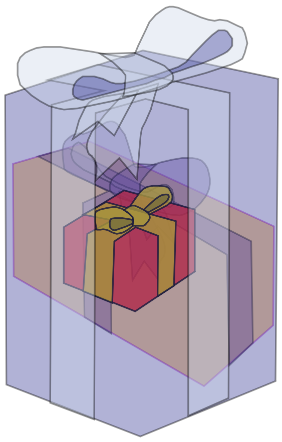
Do now as a warm-up:

Suppose someone gave you a present and that it was in a nicely wrapped box, and that box was inside yet another nicely wrapped box.



Simple as it is, think about instructions detailing how you would get to the present, if you want to do so in a polite manner that would impress and demonstrate gratitude.

2.4 The Chain Rule

ex. Compose these functions to find $g(f(x))$ $= \sqrt{\sin x}$
if $f(x) = \sin x$
and $g(x) = \sqrt{x}$

Decompose:

ex. If $h(x) = (4x^3 - x^2)^2$, find two functions f and g so that $h(x) = f(g(x))$.

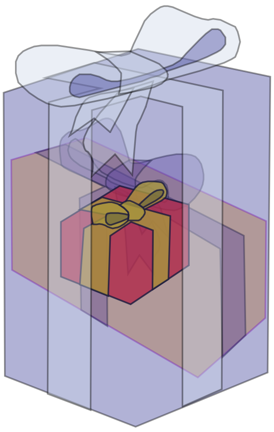
$f(x) = x^2$

$g(x) = 4x^3 - x^2$

Ogres have layers!

Onions have layers!

Matryoshka (nesting dolls) have layers!



Nested presents have layers!

Composite functions have layers!

ex. Compose these functions to find g(f(x))
if $f(x)=3x^2+1$
and $g(x)=\sqrt{x}$

$$g(f(x))=\sqrt{3x^2+1}$$

Find y':

$$y = \boxed{\sin\left(\boxed{x^2}\right)} \qquad y' = \boxed{\cos\left(\boxed{x^2}\right)} \cdot \boxed{2x}$$

$$x^{\frac{1}{2}} \to \frac{1}{2}x^{-\frac{1}{2}}$$

$$y = \boxed{\left(\boxed{\cos\left(\boxed{\sqrt{x}}\right)}\right)^4} \qquad y' = \boxed{4\left(\boxed{\cos\left(\boxed{\sqrt{x}}\right)}\right)^3} \cdot \boxed{-\sin\left(\boxed{\sqrt{x}}\right)} \cdot \boxed{\frac{1}{2\sqrt{x}}}$$

$$y = (\cos\sqrt{x})^4 \qquad y' = 4(\cos\sqrt{x})^3 \cdot (-\sin\sqrt{x}) \cdot \left(\frac{1}{2\sqrt{x}}\right)$$

$$\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$$

Thm. The Chain Rule
If y=f(m) is a differentiable function of m and m=g(x) is a differentiable function of x, then y= f(g(x)) is a differentiable function of x and

$$\frac{dy}{dx} = \frac{dy}{dm} \bullet \frac{dm}{dx}$$

$$\frac{d}{dx}\left[f\left(g(x)\right)\right] = f'\left(g(x)\right) \bullet g'(x)$$

Take the derivative of a composite function like you would unwrap a present-- one package at a time and from the outside toward the inside!

ex. $h(x) = (x^2-2x)^3$

$h'(x) = 3(x^2-2x)^2 \cdot (2x-2)$

ex. $h(x) = \sqrt{3x^2+1} = (3x^2+1)^{\frac{1}{2}}$

$h'(x) = \frac{1}{2}(3x^2+1)^{-\frac{1}{2}} \cdot (6x)$

$= \frac{1(\cancel{6}^{3}x)}{\cancel{2}_{1}\sqrt{3x^2+1}} = \frac{3x}{\sqrt{3x^2+1}}$

Thm. The General Power Rule

If $y=[u(x)]^n$ where u is a differentiable function of x, then

$$\frac{dy}{dx} = n[u(x)]^{n-1} \bullet \frac{du}{dx} \qquad \text{or} \qquad \frac{d}{dx}[u^n] = n(u)^{n-1} \bullet u'$$

$u'(x)$

ex. $g(x) = \sin^2 x$ $= (\sin x)^2$

$g'(x) = \underline{2\sin x} \cdot \cos x$

ex. $g(x) = \cos(\sin x)$

$g'(x) = -\sin(\sin x) \cdot \cos x$

ex. $\frac{d}{dx}[\tan(\sin x)] = \sec^2(\sin x) \cdot \cos x$

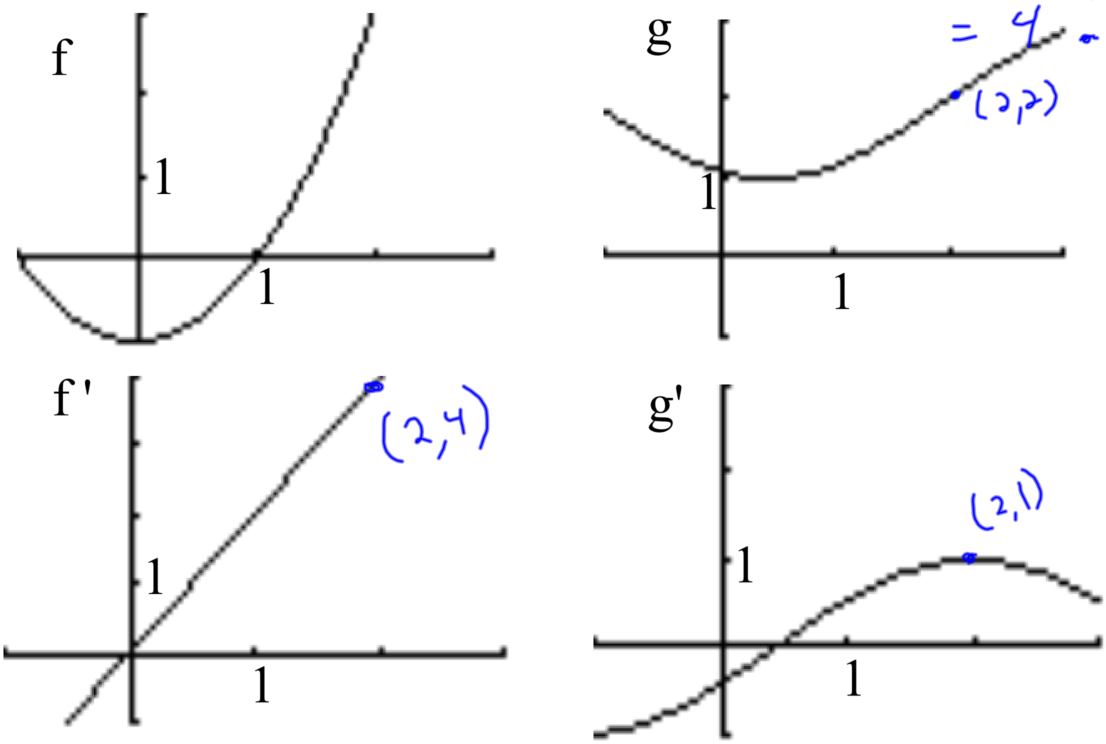
ex. Suppose that h(x)=f(g(x)) and that f '(3)=2, g(5)=3, and g'(5)=7. Find h'(5).

$$h'(x) = f'(g(x)) \cdot g'(x)$$
$$h'(5) = f'(g(5)) \cdot g'(5)$$
$$= f'(3) \cdot 7$$
$$= 2 \cdot 7 = 14$$

ex. Suppose h(x)=f(g(x)). Find h'(2).

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(2) = f'(g(2)) \cdot g'(2)$
$= f'(2) \cdot 1$
$= 4 \cdot 1 = 4$



ex. Find the line that is tangent to the graph of y= $\sqrt[5]{3x^3+4x}$ $= (3x^3+4x)^{\frac{1}{5}}$

at the point (2,2).

$y' = \frac{1}{5}(3x^3+4x)^{-\frac{4}{5}}(9x^2+4)$

$m = \frac{1}{5}(3\cdot 8 + 8)^{-\frac{4}{5}}(36+4)$

$= \frac{1}{5}(32)^{-4/5}(40) = \frac{1}{16}\cdot 8 = \frac{1}{2}$

$y = mx+b$

$2 = \frac{1}{2}\cdot 2 + b$

$2 = 1 + b$

$1 = b$

$y = \frac{1}{2}x + 1$

$\boxed{y = \frac{1}{2}(x-2)+2}$

$y - y_1 = m(x-x_1) + y_1$

ex. If $y = \sin(|x|)$, Look at a graph as a first step.

$\sin(-x)$ $\sin x$

find the derivative for all x where this function is differentiable.

$$y' = \begin{cases} \cos x & x > 0 \\ \cos(-x)\cdot(-1) & x < 0 \end{cases}$$

True or False?

The derivative of an odd function is even and the derivative of an even function is odd.

True, now prove it!

even

$f(x) = f(-x)$

$f'(x) = f'(-x)(-1)$

$f'(x) = -f'(-x)$

Odd function: f(-x) = -f(x)
Take the derivative:
[f '(-x)][-1] = -f '(x)
so f '(-x) = f '(x)
Thus, f ' is an even function.

odd

$f(-x) = -f(x)$

$(-1)f'(-x) = -f'(x)$

$f'(-x) = f'(x)$