Do now as a warm-up:

For these, y is a function explicitly defined in terms of x.

$y=\sqrt{x^2+2}$

$y=3x^2+2x$

$y=x+1$

$y=\sin x$

For these, y is a function implicitly defined in terms of x.

$y=\cos xy$

$xy=12$

$3x^2y+2y^2-5=0$

$x+xy+y=5$

Be able to explain what makes a function implicitly defined vs. explicitly defined.

2.5 Implicit Differentiation

ex. $\frac{d}{dx}[y] = \frac{dy}{dx}$ (or y')

ex. $\frac{d}{dx}[2y] = 2\frac{dy}{dx}$ $2y'$

ex. $\frac{d}{dx}[y^2] = 2y\frac{dy}{dx}$ $2yy'$

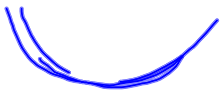
<u>Implicit Differentiation</u>

Take the derivative of each term with respect to x and solve for dy/dx.

ex. Find dy/dx for $y^2+3x=x^2$

$$y^2 = x^2 - 3x$$

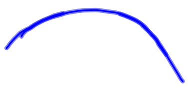
$$y = \pm\sqrt{x^2-3x}$$



$$2y\frac{dy}{dx} + 3\cancel{\frac{dx}{dx}} = 2x$$

1

$-3 \qquad -3$



$$2y\frac{dy}{dx} = 2x-3$$

$$\frac{dy}{dx} = \frac{2x-3}{2y}$$

ex. Find dy/dx for sinx+cosy=2y

$$\cos x - \sin y \cdot \frac{dy}{dx} = 2\frac{dy}{dx}$$

$$\cos x = 2\frac{dy}{dx} + \sin y \frac{dy}{dx}$$

$$\cos x = \frac{dy}{dx}(2+\sin y)$$

$$\frac{\cos x}{2+\sin y} = \frac{dy}{dx}$$

ex. Find dy/dx for $y^2+xy+x^2=5$

$$2y\frac{dy}{dx} + 1y + x\frac{dy}{dx} + 2x = 0$$

$$\frac{dy}{dx}(2y+x) = -y-2x$$

$$\frac{dy}{dx} = \frac{-y-2x}{2y+x}$$

ex. $x^2 - xy + y^2 = 3$

a. Show that $\dfrac{dy}{dx} = \dfrac{y-2x}{2y-x}$

b. Find any point(s) where the tangent line is horizontal.

a. $2x - 1y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2y - x) = y - 2x$

$\frac{dy}{dx} = \dfrac{y-2x}{2y-x}$

b. $y - 2x = 0$

$y = 2x$

$x^2 - 2x^2 + 4x^2 = 3$

$3x^2 = 3$

$x = \pm 1$

$(1, 2)$

$(-1, -2)$

ex. Find $\frac{d^2y}{dx^2}$ for $1-xy=x-y$

$$-1y - x\frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$-y-1 = \frac{dy}{dx}(x-1)$$

$$\frac{-y-1}{x-1} = \frac{dy}{dx}$$

$$\frac{-\frac{dy}{dx}(x-1) - (-y-1)(1)}{(x-1)^2} = \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{-\left(\frac{-y-1}{\cancel{x-1}}\right)(\cancel{x-1}) + y + 1}{(x-1)^2}$$

$$= \frac{y+1+y+1}{(x-1)^2} = \frac{2y+2}{(x-1)^2}$$

ex. Find where xy=16 has a horizontal tangent line.

$$1y + x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

$$-y = 0$$

$$y = 0$$

$$x \cdot 0 = 16$$

$$0 = 16$$

no horizontal tangent lines