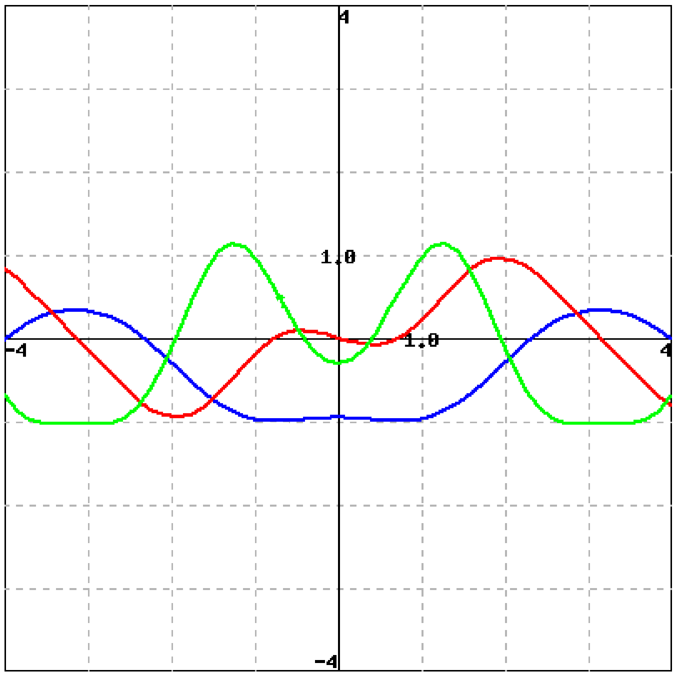
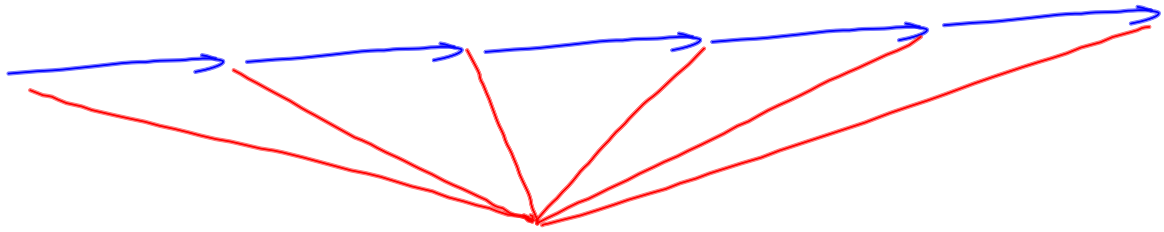
Do now as a warm-up:

A quick review:

This graph shows a function, its derivative, and its second derivative.

Which is which and how can you tell?





# 2.6 Related Rates

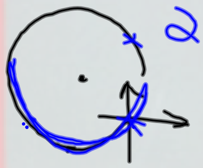
ex. Cheryl is drawing a circle on an Etch-A-Sketch®:

$$\frac{d}{dt}\left[x^2 + y^2 = 4\right] \qquad 1^2 + y^2 = 4 \qquad y^2 = 3 \qquad y = \pm\sqrt{3}$$

At the spot where x=1, she's turning the knob so that the stylus' horizontal rate of change is 2 inches/minute. $\leftarrow \frac{dx}{dt}$

How fast must the stylus' vertical rate of change be to accurately draw the curve?

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

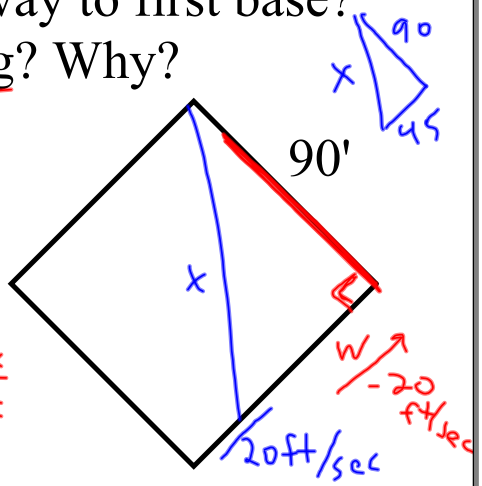


$$2\cdot 1\cdot 2 + 2\cdot -\sqrt{3}\,\frac{dy}{dt} = 0$$

$$4 - 2\sqrt{3}\,\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-4}{-2\sqrt{3}} = \frac{2}{\sqrt{3}} \text{ in./min.}$$

ex. A baseball diamond is a square 90 feet on each side. A runner starts from home plate towards first base at 20ft/sec. How fast is the runner's distance from second base changing when the runner is halfway to first base? Is this distance increasing or decreasing? Why?




$$\frac{d}{dt}\left[w^2 + 90^2 = x^2\right]$$

$$2w\frac{dw}{dt} + 0 = 2x\frac{dx}{dt}$$

$$2\cdot 45\text{ft}\left(\frac{-20\text{ ft}}{\text{sec}}\right) = 2\sqrt{10125}\text{ ft}\,\frac{dx}{dt}$$

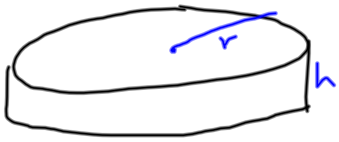
$$-8.944 \approx \frac{dx}{dt}$$

decr. since this is neg.

ex. A tanker spilled oil in a bay. Oil-consuming bacteria are deployed that remove oil at a rate of 5 $ft^3$/hr. The oil slick is cylindrical (the thickness of the slick is the height). When the radius of the slick is 500 ft, the thickness is 0.01 ft, and is decreasing at a rate of 0.001 ft/hr. At this time, what is the rate at which the area of the slick is changing?

$V = \pi \cdot 500^2 \cdot .01$

$= 2500\pi \ ft^3$




$\frac{dh}{dt} = -.001 \ ft/hr$ $\quad V = \pi r^2 h$

$\frac{dV}{dt} = -5$

$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$

$-5 = 2\pi \cdot 500 \frac{dr}{dt} \cdot .01 + \pi (500)^2 (-.001)$

$-5 = 31.415927 \frac{dr}{dt} - 785.398163$

$\frac{dr}{dt} = 24.840845$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(500)(24.8408)$

$= 78039.675 \ ft^2/hr$

ex. A spherical balloon is being inflated at a rate of 4 cubic inches per minute. Find the rate of change of the radius when the surface area is 64π square inches.

$\frac{dV}{dt} = 4 \frac{in^3}{min}$

$\frac{dr}{dt} = ? \frac{in}{min}$

$\frac{64\pi}{4\pi} = SA = \frac{4\pi r^2}{4\pi}$

$16 = r^2$

$r = 4 \text{ in}$

$V = \frac{4}{3}\pi r^3$

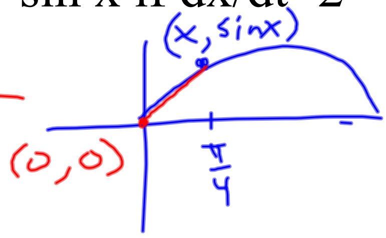
$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$4 \frac{in^3}{min} = 4\pi 4^2 in^2 \frac{dr}{dt}$

$\frac{4 \frac{in^3}{min}}{64\pi in^2} = \frac{64\pi \frac{dr}{dt} in^2}{64\pi in^2}$

$\frac{1}{16\pi} \frac{in}{min} = \frac{dr}{dt}$

ex. Find the rate of change of the distance between the origin and a fly crawling on the graph of y=sin x if dx/dt=2 cm/sec when x=π/4.




$$d=\sqrt{(x-0)^2+(\sin x-0)^2}$$

$$d=\sqrt{x^2+(\sin x)^2}$$

$$d^2=x^2+(\sin x)^2$$

$$\cancel{2}d\frac{dd}{dt}=\cancel{2}x\frac{dx}{dt}+\cancel{2}\sin x\cos x\frac{dx}{dt}$$

$$d=\sqrt{\left(\frac{\pi}{4}\right)^2+\left(\sin\frac{\pi}{4}\right)^2}$$

$$d=\sqrt{\left(\frac{\pi}{4}\right)^2+\frac{1}{2}}$$

$$\sqrt{\left(\frac{\pi}{4}\right)^2+\frac{1}{2}}\,\frac{dd}{dt}=\frac{\pi}{4}\cdot 2+\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}\cdot 2$$

$$\frac{dd}{dt}=\frac{\frac{\pi}{2}+\frac{2}{4}\cdot 2}{\sqrt{\left(\frac{\pi}{4}\right)^2+\frac{1}{2}}}$$

$$\frac{dd}{dt}=\frac{\frac{\pi}{2}+1}{\sqrt{\left(\frac{\pi}{4}\right)^2+\frac{1}{2}}}$$