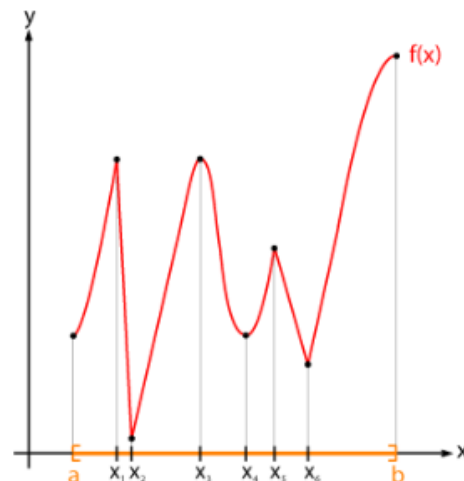


Do now as a warm-up:

Suppose this graph is a function  $f$ , defined on  $[a,b]$ . What would you say about the value of  $f$  at each of these  $x$  values:  $a$ ,  $x_1$ ,  $x_2$ ,  $x_3$ ,  $x_4$ ,  $x_5$ ,  $x_6$ , and  $b$ ? What would you say about the value of  $f'$  at these  $x$  values:  $x_1$ ,  $x_2$ ,  $x_3$ ,  $x_4$ ,  $x_5$ , and  $x_6$ ?



### 3.1 Extrema on an interval

Defn. Let  $f$  be defined on an interval  $I$  containing  $c$ .

1.  $f(c)$  is the minimum on  $I$  if  $f(c) \leq f(x)$  for all  $x$  in  $I$ .
2.  $f(c)$  is the maximum on  $I$  if  $f(c) \geq f(x)$  for all  $x$  in  $I$ .

extreme values or extrema on an interval

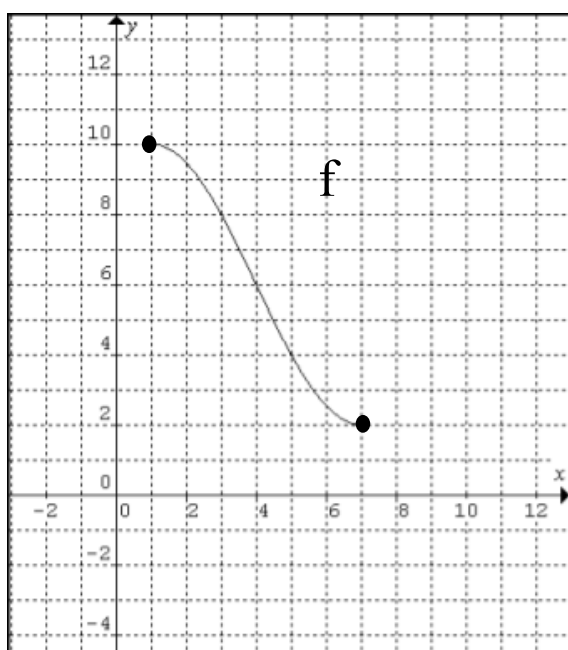
absolute minimum on the interval

absolute maximum on the interval

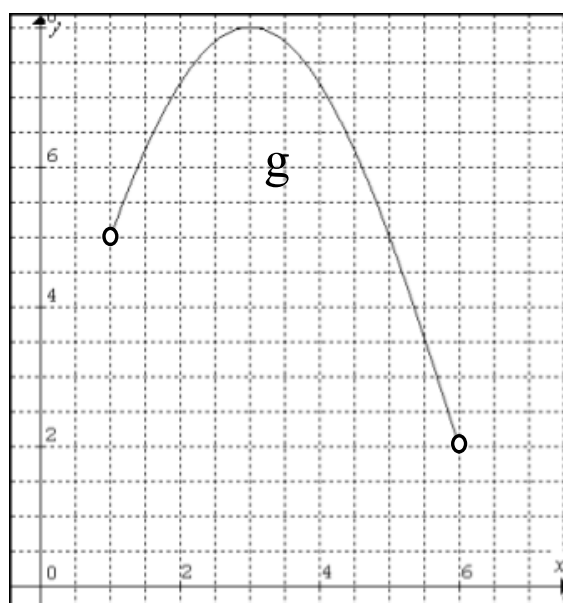
Thm.

$f$  is cont's on  $[a,b]$

$f$  has a maximum and a minimum

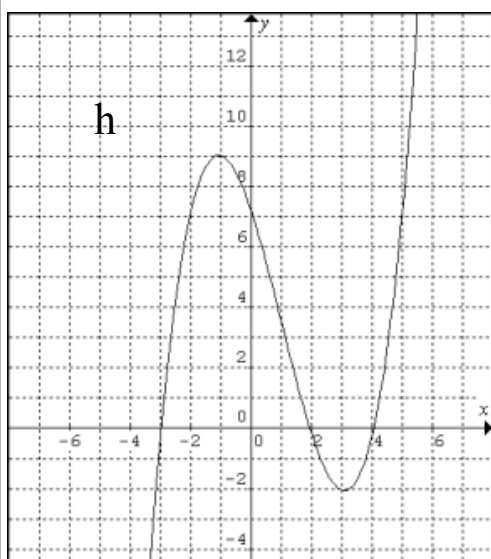


ex.  $f$  is defined on  $[1,7]$ .  
 The max is 10 & it occurs at  $x=1$ .  
 The min is 2 & it occurs at  $x=7$ .



ex.  $g$  is defined on  $(1,6)$ .  
 The max is 8 & it occurs at  $x=3$ .  
 $g$  has no min.

Defn. If  $f(c)$  is a max on an open interval containing  $c$ , then  $f(c)$  is a relative (local) maximum. If  $f(c)$  is a min, then  $f(c)$  is a relative (local) minimum. **Relative (local) extrema happen only at critical numbers.**



ex. At  $x = -1$ ,  $h$  has a relative (local) maximum of 9 & at  $x = 3$  there is a relative (local) minimum of -2.

$h$  has two critical numbers, -1 and 3.

Defn. Let  $f(c)$  be defined. If  $f'(c) = 0$  or  $f'(c)$  is undefined, then  $c$  is a critical number.

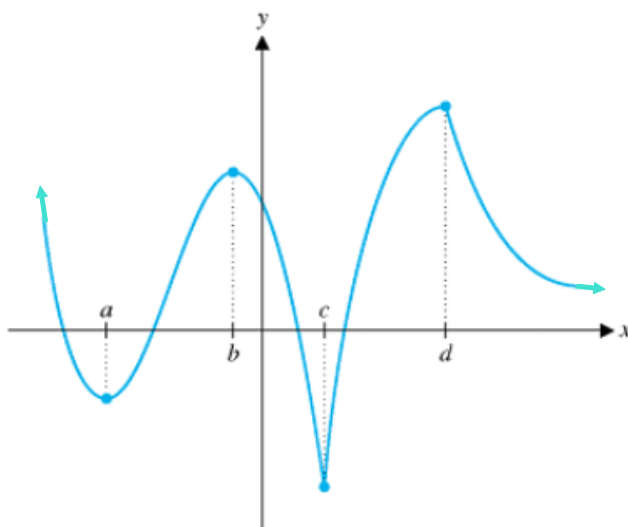
Thm.

at  $x=c$ , there's a  
local max or min



$c$  is a critical number

ex. Find the critical values of this function that is defined on all reals. At each critical value, identify whether it is a local or absolute extrema and discuss the value of the derivative.



ex. Find the critical values of

$$f(x) = x^3 + 3x^2 - 9x + 2$$

ex. Find all extrema for  
 $f(x) = 3x^4 - 4x^3$   
on  $[-1, 2]$ .



ex. Use a calculator to find all extrema of the function

$$f(x) = x^2 + 4x + 9\cos x$$

on  $[-6,4]$

ex. Find the value of  $a$  so that

$$f(x) = ax^2 + 14x - 5$$

has a local extreme value at  $x=1$ .