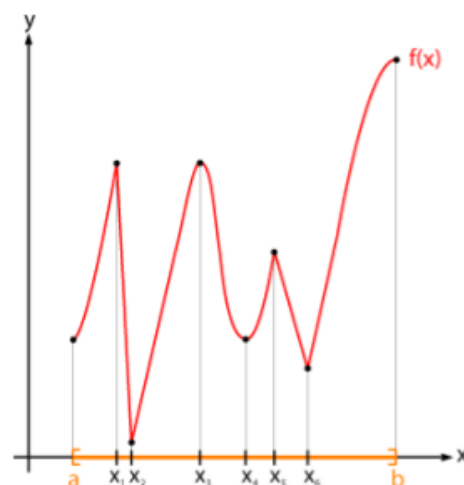


Do now as a warm-up:

Suppose this graph is a function f , defined on $[a,b]$. What would you say about the value of f at each of these x values: a , x_1 , x_2 , x_3 , x_4 , x_5 , x_6 , and b ? What would you say about the value of f' at these x values: x_1 , x_2 , x_3 , x_4 , x_5 , and x_6 ?



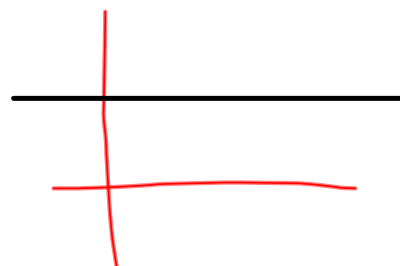
3.1 Extrema on an interval

Defn. Let f be defined on an interval I containing c .

1. $f(c)$ is the minimum on I if $f(c) \leq f(x)$ for all x in I .
2. $f(c)$ is the maximum on I if $f(c) \geq f(x)$ for all x in I .

max is y , it occurs at x

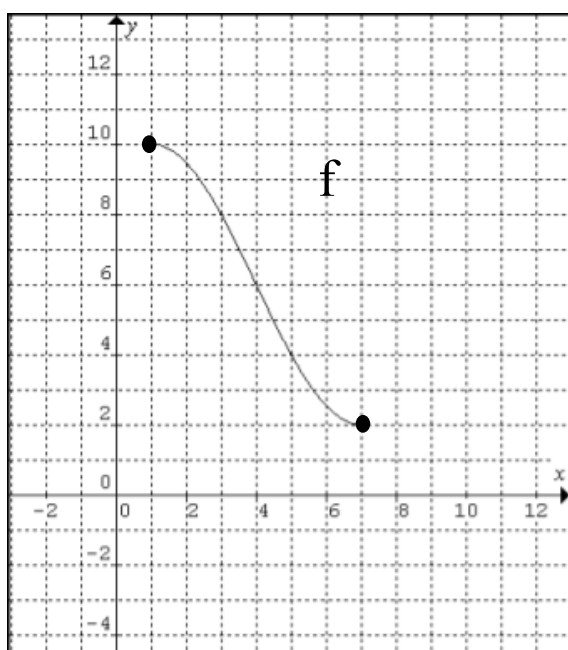
extreme values or extrema on an interval
absolute minimum on the interval
absolute maximum on the interval



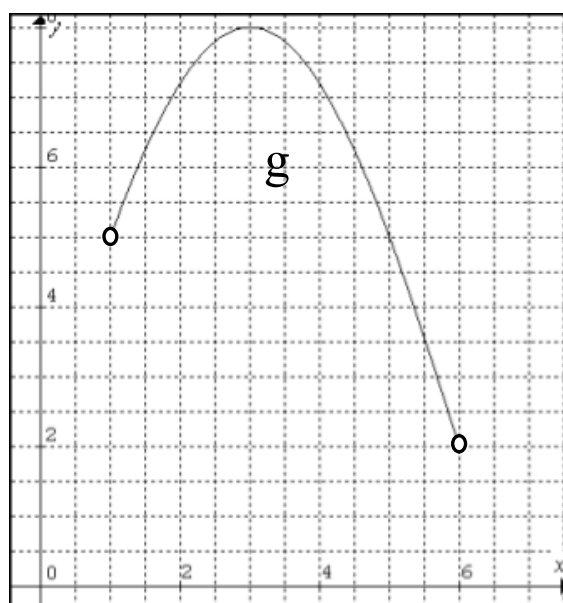
Thm.

f is cont's on $[a,b]$

f has a maximum and a minimum

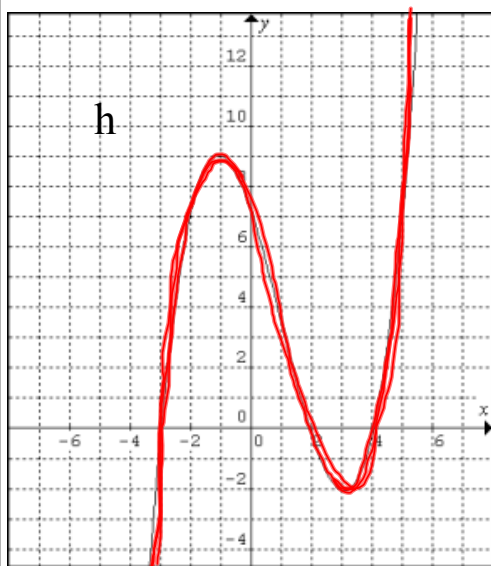


ex. f is defined on $[1,7]$.
 The max is 10 & it occurs at $x=1$.
 The min is 2 & it occurs at $x=7$.



ex. g is defined on $(1,6)$.
 The max is 8 & it occurs at $x=3$.
 g has no min.

Defn. If $f(c)$ is a max on an open interval containing c , then $f(c)$ is a relative (local) maximum. If $f(c)$ is a min, then $f(c)$ is a relative (local) minimum. **Relative (local) extrema happen only at critical numbers.**



ex. At $x=-1$, h has a relative (local) maximum of 9 & at $x=3$ there is a relative (local) minimum of -2.

h has two critical numbers, -1 and 3.

D	R
x	y
c	e

Defn. Let $f(c)$ be defined. If $f'(c) = 0$ or $f'(c)$ is undefined, then c is a critical number.

Thm.

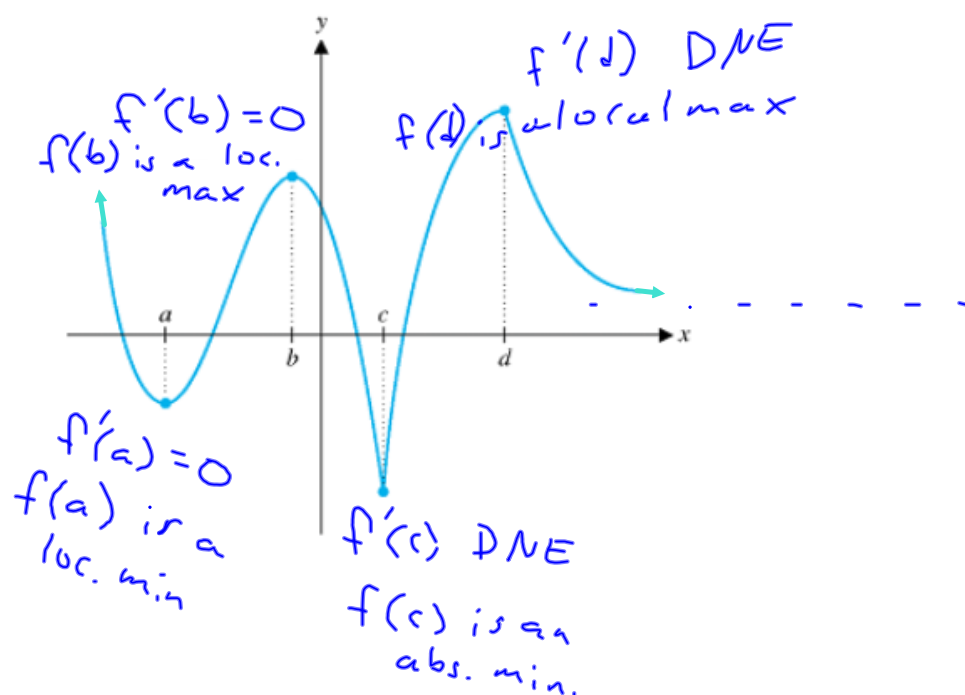
at $x=c$, there's a
local max or min

always

c is a critical number

maybe

ex. Find the critical values of this function that is defined on all reals. At each critical value, identify whether it is a local or absolute extrema and discuss the value of the derivative.



ex. Find the critical values of

$$f(x) = x^3 + 3x^2 - 9x + 2$$

$$f'(x) = 3x^2 + 6x - 9$$

$$3x^2 + 6x - 9 = 0$$

$$x^2 + 2x - 3 = 0$$

$x = 1, -3$ are critical #s.

$$f'(x) = 0$$

$$x = -3, 1$$

are crit. #s.

ex. Find all extrema for

$$f(x) = 3x^4 - 4x^3$$

on $[-1, 2]$.

$$f'(x) = 12x^3 - 12x^2$$

$$x^2(x-1) = 0$$

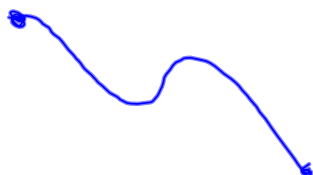
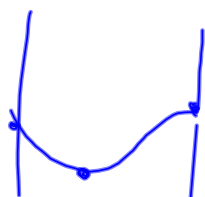
$$\text{crit \#s: } x=0, x=1$$

$$f(-1) = 7$$

$$f(0) = 0$$

$$f(1) = -1 \begin{matrix} \text{abs.} \\ \text{min} \end{matrix}$$

$$f(2) = 48 - 32 = 16 \begin{matrix} \text{abs.} \\ \text{max} \end{matrix}$$



ex. Use a calculator to find all extrema of the function

$$f(x) = x^2 + 4x + 9\cos x$$

on $[-6, 4]$

$$f(-6) = 20.642$$

$$f(x_1) = -11.934 \text{ min}$$

$$f(x_2) = 10.189$$

$$f(x_3) = 8.252$$

$$f(4) = 26.117 \text{ max}$$

$$f'(x) = 2x + 4 - 9\sin x$$

$$f'(x) = 0$$

$$\text{when } x_1 = -2.933$$

$$\text{and } x_2 = .622$$

$$\text{and } x_3 = 2.031$$

ex. Find the value of a so that

$$f(x) = ax^2 + 14x - 5$$

$$f'(x) = 2ax + 14$$

has a local extreme value at $x=1$.

$$f'(1) = 0$$

$$2a \cdot 1 + 14 = 0$$

$$2a = -14$$

$$a = -7$$