

Do now as a warm-up:

1. Plot 2 points that have the same y value.
2. Connect those points with a graph that is continuous and differentiable. (Your graph should be smooth and have no jumps, breaks, holes, gaps, nor vertical asymptotes.)
3. Count the number of places between your original 2 points that would have a horizontal tangent line.
4. Repeat steps 1-3 until you think you could make a conjecture about what is always true in this situation.

3.2 Rolle's Theorem and the Mean Value Theorem (MVT).

Rolle's Thm.

f is cont's on $[a,b]$
 f is diff 'ble on (a,b)
 $f(a)=f(b)$

at least one $c \in (a,b)$
where $f'(c)=0$.

Note: if any of these 3
hypotheses/premises fail,
we can't apply Rolle's Thm.

ex. If possible, find the value of c that satisfies Rolle's Thm. if $f(x) = x^2 - 2x$ for the interval $[0, 2]$.

ex. Use Rolle's Thm. to show that

$$f(x) = x^5 + 10x - 5$$

has at most 1 real root.



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ex. For the function $f(x) = |\sin x|$

$f(-1)=f(1)$, but there is no number c in $(-1,1)$ such that $f'(c)=0$. Why does this not contradict Rolle's Thm.?

Before you answer that question,
what does your calculator say $f'(0)=?$

Try this!

1. Sketch a graph that is continuous and differentiable between any 2 points. (Your graph should be smooth and have no jumps, breaks, holes, gaps, nor vertical asymptotes.)
2. Connect your endpoints with a line
3. Count the number of places between your original 2 points that would have a tangent line parallel to the line you drew in step 2.
4. Repeat steps 1-3 until you think you could make a conjecture about what is always true in this situation.

Mean Value Thm. (MVT)

f is cont's on $[a,b]$
f is diff 'ble on (a,b)



at least one $c \in (a,b)$
where
$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Note: if either of these 2 hypotheses/premises fail, we can't apply the MVT.

ex. If possible, apply the MVT to $f(x) = x^{\frac{2}{3}}$
on the interval $[0,1]$.

ex. Suppose a driver enters a tollway at mile marker 110, picking up a card at the toll plaza, and exits the tollway via the toll plaza at mile marker 215 just 1 hour and 16 minutes later. The toll booth attendant collects the \$4.50 toll and issues a speeding ticket. How does the MVT tell us that the driver was speeding?

SPEED FINE CALCULATOR	
1. ENTER YOUR SPEED FROM THE TICKET	83
2. ENTER THE SPEED LIMIT FROM THE TICKET	65
3. DETERMINE COURT COSTS Look at ticket # in the upper right corner of ticket. Each ticket has a letter followed by a series of numbers. Place this in the box .	A12345
4. <input type="button" value="Calculate Fine"/>	<input type="checkbox"/> Citation issued in a Construction Zone
MPH OVER	18
SPEEDING FINE:	\$ 75
COURT COSTS:	\$ 99
YOUR TOTAL FINE IS:	\$ 174
<input type="button" value="Clear"/>	Eff. 9/23/08 The Court does NOT accept payments by telephone.

These are actual exit numbers on the Ohio Turnpike.

I just made up some number and put it in some Ohio county court website.

BTW, clicking this doubles the fine to \$150.

ex. Why doesn't the MVT apply to...

$$f(x) = \frac{x^2 + 1}{x} \quad \text{on the interval } [-1, 2]?$$

$$f(x) = |x^2 - 4x| \quad \text{on the interval } [-1, 1]?$$

ex. Does there exist a differentiable function f with $f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 0$ for all x ?

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