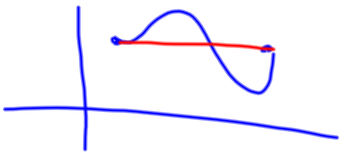
Do now as a warm-up:

1. Plot 2 points that have the same y value.
2. Connect those points with a graph that is continuous and differentiable. (Your graph should be smooth and have no jumps, breaks, holes, gaps, nor vertical asymptotes.)
3. Count the number of places between your original 2 points that would have a horizontal tangent line.
4. Repeat steps 1-3 until you think you could make a conjecture about what is always true in this situation.

3.2 Rolle's Theorem and the Mean Value Theorem (MVT).

Rolle's Thm.

f is cont's on [a,b] f is diff 'ble on (a,b) f(a)=f(b)	$\Rightarrow$	$\exists$ at least one c$\in$(a,b) where f '(c)=0.



Note: if any of these 3 hypotheses/premises fail, we can't apply Rolle's Thm.

ex. If possible, find the value of c that satisfies Rolle's Thm. if f(x)= $x^2 - 2x$ for the interval [0,2].

Since f is a poly.
it's cont's & diff'ble.

$f(0) = 0$

$f(2) = 0$

So Rolle's Thm applies

$f'(x) = 2x - 2$

$0 = 2c - 2$

$1 = c$

ex. Use Rolle's Thm. to show that

$$f(x) = x^5 + 10x - 5$$

has at most 1 real root.

We'll argue by contradiction.
One assumption: f has 2 roots, a and b.
Then f(a)=f(b)...

$f(a) = 0 = f(b)$

$f'(x) = 5x^4 + 10$

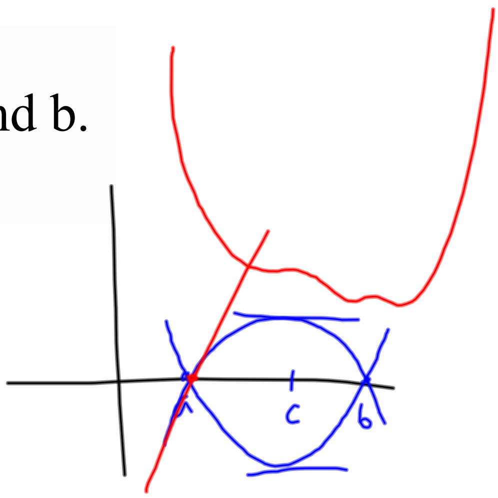
Then $5x^4 + 10 = 0$

$x^4 + 2 = 0$

$x^4 = -2$ which has no real solutions.

So our assumption is false.

f has fewer than 2 roots.

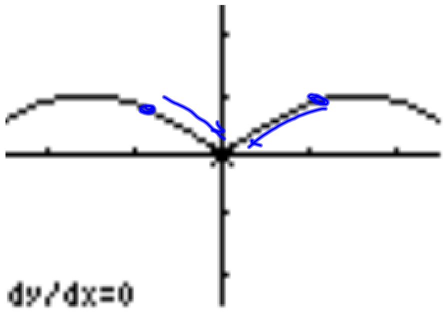


ex. For the function $f(x) = |\sin x|$

f(-1)=f(1), but there is no number c in (-1,1) such that f '(c)=0. Why does this not contradict Rolle's Thm.?

f is not diff'ble at x = 0, Rolle's doesn't apply

Before you answer that question,
what does your calculator say f '(0)=?



Try this!

1. Sketch a graph that is continuous and differentiable between any 2 points. (Your graph should be smooth and have no jumps, breaks, holes, gaps, nor vertical asymptotes.)
2. Connect your endpoints with a line
3. Count the number of places between your original 2 points that would have a tangent line parallel to the line you drew in step 2.
4. Repeat steps 1-3 until you think you could make a conjecture about what is always true in this situation.

Mean Value Thm. (MVT)

f is cont's on [a,b]

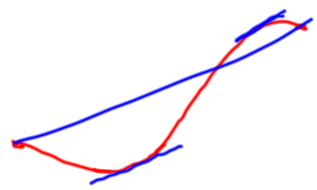
f is diff 'ble on (a,b)

$\Rightarrow$

$\exists$ at least one $c \in (a,b)$ where

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

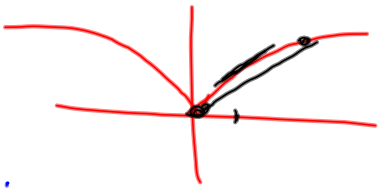
Note: if either of these 2 hypotheses/premises fail, we can't apply the MVT.



ex. If possible, apply the MVT to $f(x) = x^{\frac{2}{3}}$ on the interval [0,1].

$f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$

$f'(c) = \dfrac{f(1) - f(0)}{1 - 0}$



is conts on [0, 1]

is diff'ble on (0, 1)

$\frac{2}{3} c^{-\frac{1}{3}} = \dfrac{1 - 0}{1 - 0}$

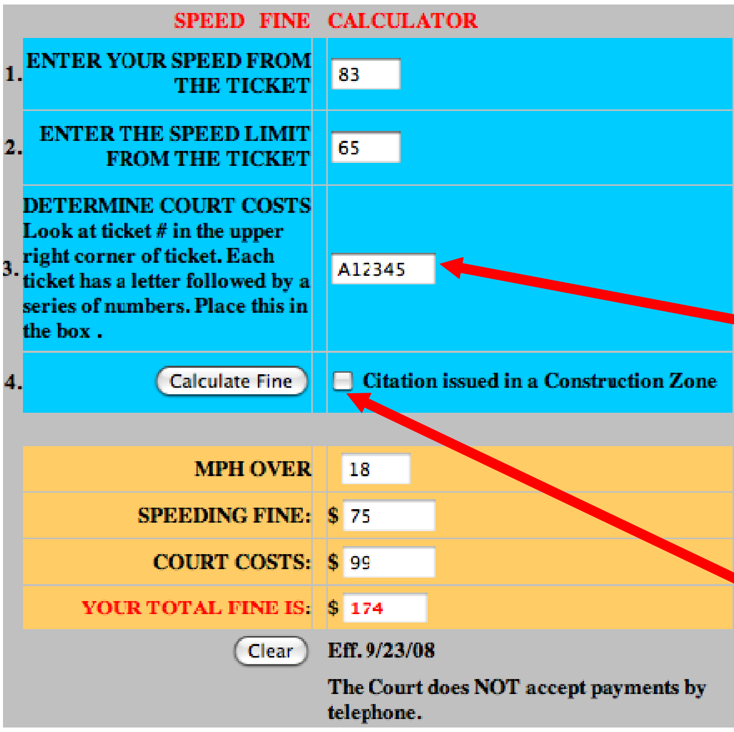
$\frac{3}{2} \cdot \dfrac{2}{3\sqrt[3]{c}} = 1 \cdot \frac{3}{2}$

$\dfrac{1}{\sqrt[3]{c}} = \dfrac{3}{2}$

$\left(\sqrt[3]{c}\right)^3 = \left(\frac{2}{3}\right)^3$

$c = \frac{8}{27}$

ex. Suppose a driver enters a tollway at mile marker 110, picking up a card at the toll plaza, and exits the tollway via the toll plaza at mile marker 215 just 1 hour and 16 minutes later. The toll booth attendant collects the $4.50 toll and issues a speeding ticket. How does the MVT tell us that the driver was speeding?



These are actual exit numbers on the Ohio Turnpike.

I just made up some number and put it in some Ohio county court website.

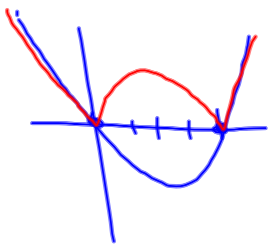
BTW, clicking this doubles the fine to $150.

ex. Why doesn't the MVT apply to...

$$f(x) = \frac{x^2 + 1}{x}$$ on the interval [-1,2]?

f is not cont's at x=0

$$f(x) = |x^2 - 4x|$$ on the interval [-1,1]?



not diff'ble at x=0 (nor at x=4)

ex. Does there exist a differentiable function
f with f(0) = -1, f(2) = 4, and f '(x)≤0 for all x?

If f is diff 'ble, it must be cont's.
Then the MVT guarantees a value
$c \in (0,2)$ such that...

no

$$f'(c) = \frac{-1-4}{0-2} = \frac{-5}{-2} = \frac{5}{2} > 0$$