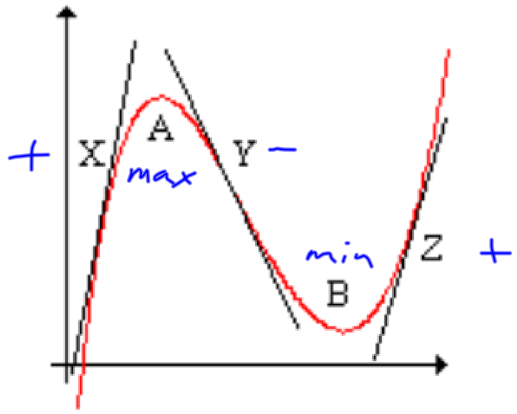
Do now as a warm-up:

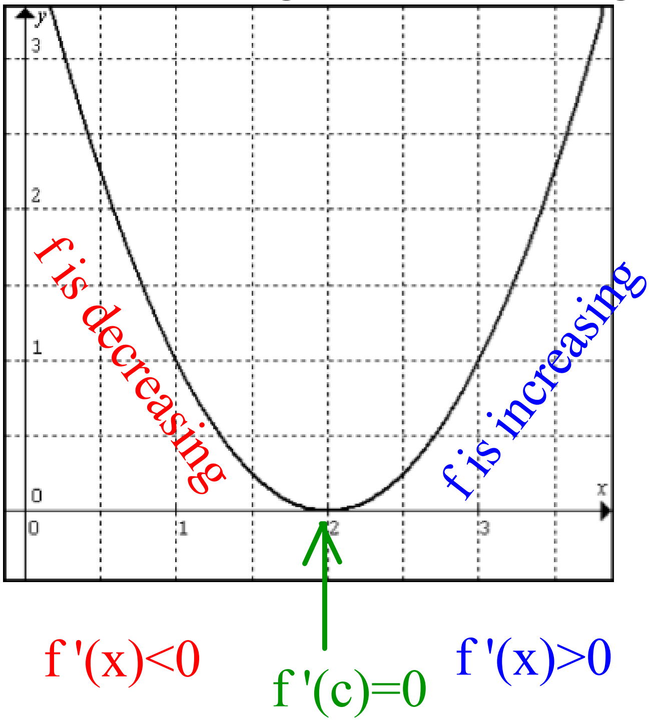
1. Classify point A and point B as either a local maximum or a local minimum.



2. Classify the slope at points X, Y, and Z as either positive or negative.

3. Devise a verbal rule that relates slope along a curve to the position of local extrema.

3.3 Increasing and Decreasing Functions



Informal Defn. If, for each pair of points on an interval, the left point is lower than the right point, the function is increasing on the interval and if the right point is lower, the function is decreasing on an interval.

Formal Defn. A function is increasing on an interval if for any numbers $x_1<x_2$ then $f(x_1)<f(x_2)$and the function is decreasing on an interval when $f(x_1)>f(x_2)$.

Thm. Test for increasing or decreasing functions

Let f be continuous on [a,b] and differentiable on (a,b).

1. If $f'(x)>0 \quad \forall x\in(a,b)$ then f is increasing on [a,b].
2. If $f'(x)<0 \quad \forall x\in(a,b)$ then f is decreasing on [a,b].
3. If $f'(x)=0 \quad \forall x\in(a,b)$ then f is constant on [a,b].

ex. Determine where the function is increasing or decreasing if $f(x) = -x^3 + 12x + 25$

f is a polynomial, so it's cont's & diff'ble everywhere

$f'(x) = -3x^2 + 12$

$-3x^2 + 12 = 0$

$x^2 - 4 = 0$

$x = \pm 2$

on $(-\infty, -2)$, $f'(x) < 0$, so $f(x)$ is decreasing.

$f'(-3) = -3(9) + 12 < 0$

on $(-2, 2)$, $f'(x) > 0$, so $f(x)$ is increasing

$f'(0) = 12 > 0$

on $(2, \infty)$, $f'(x) < 0$, so $f(x)$ is decreasing.

$f'(3) = f'(-3) < 0$

Thm. First Derivative Test (FDT)

f is continuous on I
f is differentiable on I, except possibly at c
c is a critical number
f ' changes from - to +
+ to -

f(c) is a relative min
max

ex. At what x values does f have extrema and on what intervals is f is increasing or decreasing if

$$f'(x) = (x-5)(x+3)(x-3)$$

$f'(x) = 0$ when $x = 5, -3,$ or 3

these are critical #s

On $(-\infty, -3)$, $f'(x) < 0$ so $f(x)$ is decreasing

on $(-3, 3)$, $f'(x) > 0$ so $f(x)$ is increasing

on $(3, 5)$, $f'(x) < 0$ so $f(x)$ is decreasing

On $(5, \infty)$, $f'(x) > 0$ so $f(x)$ is increasing

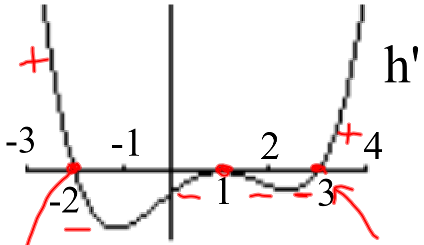
At $x = -3$, $f(x)$ has a rel. min because $f'(x)$ changes from neg. to pos. there.

At $x = 3$, $f(x)$ has a rel. max because $f'(x)$ changes from + to − at $x = 3$.

At $x = 5$, $f(x)$ has a rel. min. because $f'(x)$ changes from − to + at $x = 5$.

ex. The function h is defined on (-3,4) and the graph below is the graph of h'. Find the x coordinates of all relative maxima of h.

h' changes + to —

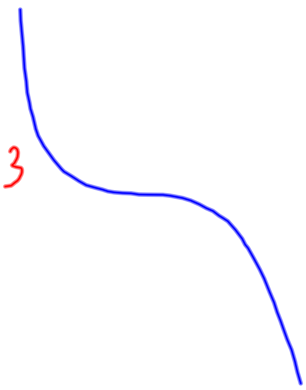


crit. #s

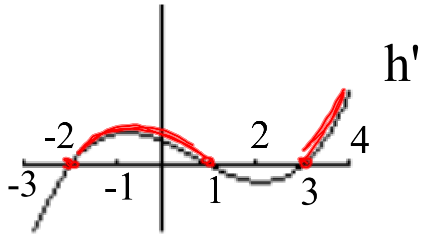
-2, 1, 3

at x = -2
h' changes
+ to —
h has a rel. max.

h has a rel. min at x = 3



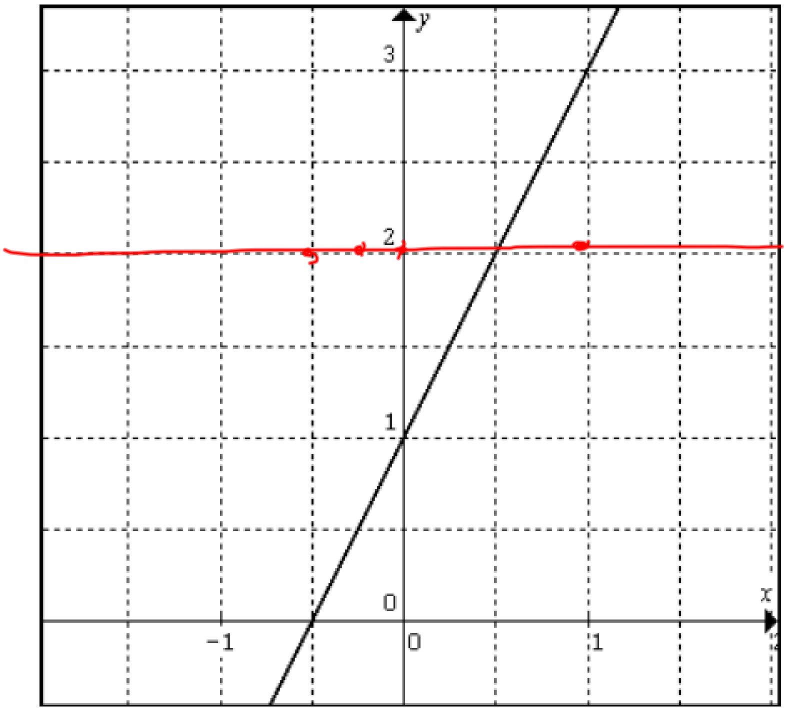
ex. The graph of h' is shown below. For what values of x is h increasing?



On $(-2, 1)$

& $(3, \infty)$ because h' is positive on these intervals.

ex. This is the graph of f. Sketch the graph of f '.



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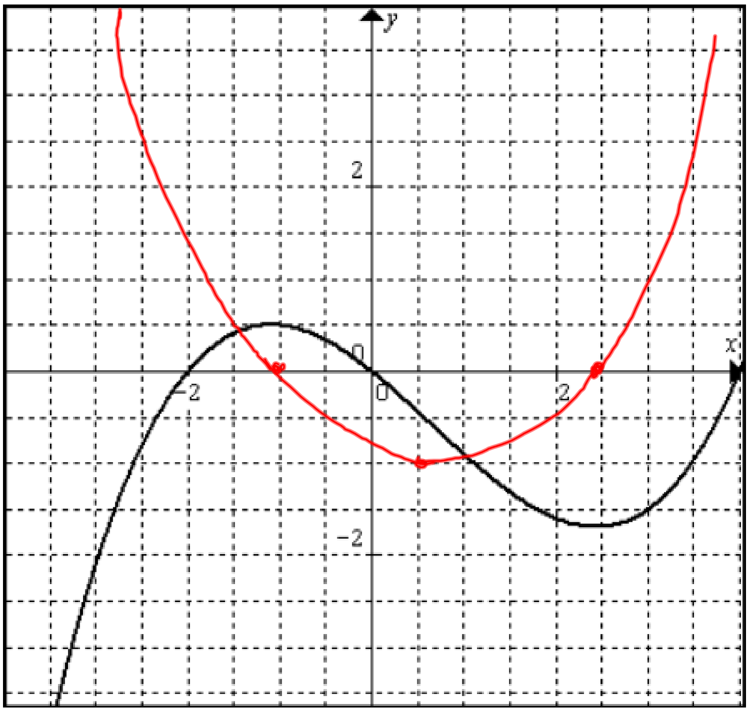
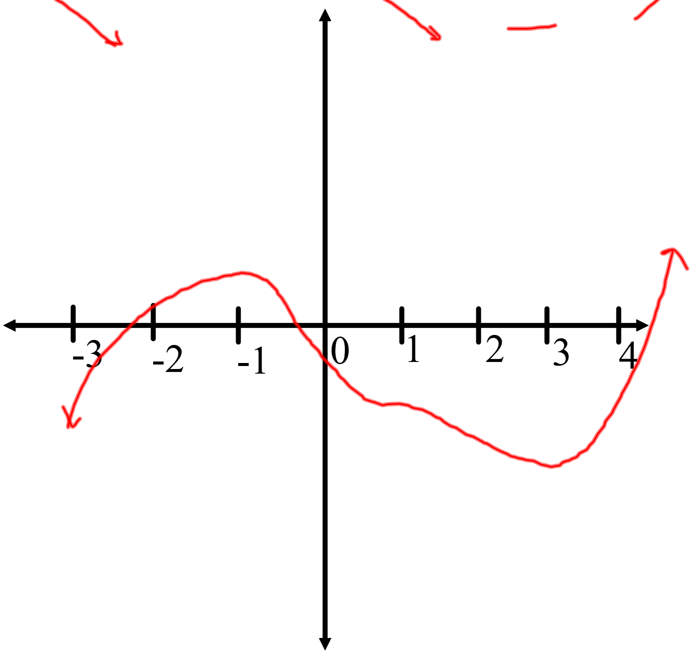


ex. Sketch the graph of a function whose derivative satisfies the following properties:

x	$(-\infty,-1)$	-1	(-1,1)	1	(1,3)	3	$(3,\infty)$
f ′(x)	Positive	0	Negative	0	Negative	0	Positive

f



ex. If g is a diff 'ble function such that g(x)<0 for all real numbers x. Find the x coordinate of all relative maxima and relative minima of the function f if $f'(x) = (x^2 - 4)g(x)$

$f'(x) = (x-2)(x+2)\,\underline{g(x)}$

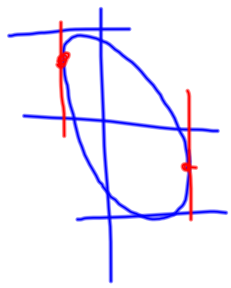
On $(-\infty,-2)$, $f'(x) < 0$
on $(-2, 2)$, $f'(x) > 0$
on $(2, \infty)$, $f'(x) < 0$

at $x = -2$, f has a rel. min because f' changes from $-$ to $+$

at $x = 2$ f has a rel. max because f' changes from $+$ to $-$.

ex. Find the highest and lowest points on the curve

$$x^2 + y^2 + xy = 12$$



$$2x + 2y\frac{dy}{dx} + 1y + x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2y + x) = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{2y + x}$$

$-2x - y = 0$

$y = -2x$

$x^2 + (-2x)^2 + x(-2x) = 12$

$x^2 + 4x^2 - 2x^2 = 12$

$3x^2 = 12$

$x = \pm 2$

$(2, -4)$

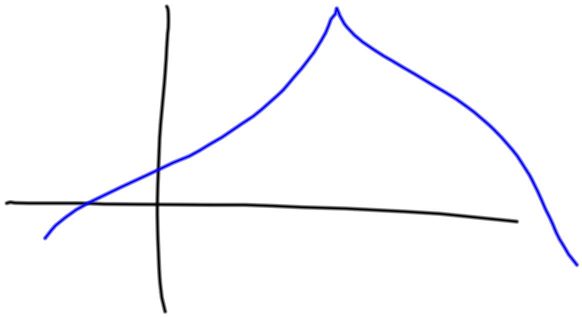
$(-2, 4)$

$2y + x = 0$

$x = -2y$

$(-2y)^2 + y^2 - 2y^2 = 12$

$3y^2 = 12$



ex. Use the first derivative to show that

$f(x) = x^4 - 4x + 2$ has at most 2 real roots.

$f'(x) = 4x^3 - 4$

$4x^3 - 4 = 0$

$x^3 - 1 = 0$

$(x-1)(x^2+x+1) = 0$

$x = 1$ x not real

$f''(x) = 12x^2 \geq 0$

f' is increasing $\forall x$

f has only 1 extrema
so f crosses the x axis
at most twice
So there are at most 2
real roots

f ' has 1 real root and is increasing

more info