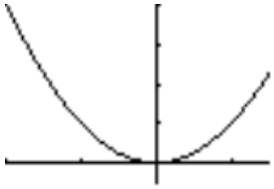
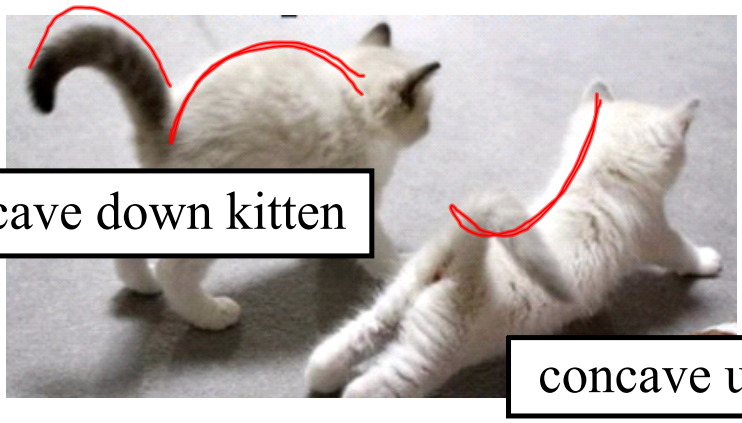
Do now as a warm-up:

1. Suppose we drew many tangent lines for this first curve. How do the slopes of these tangent lines change as we look from left to right?



2. Suppose we drew many tangent lines for this second curve. How do the slopes of these tangent lines change as we look from left to right?

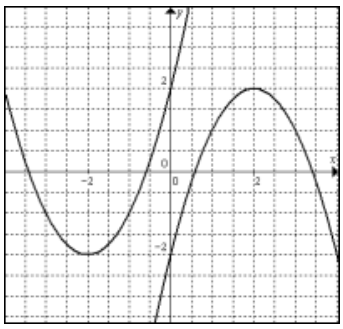
3.4 Concavity and the Second Derivative Test



concave down kitten

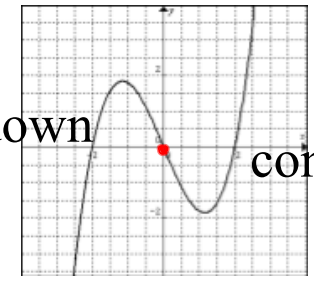
concave up kitten

concave up
(like a cup)



concave down
(like a frown)

concave down
part



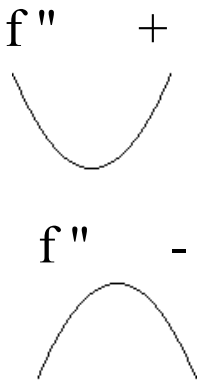
concave up part

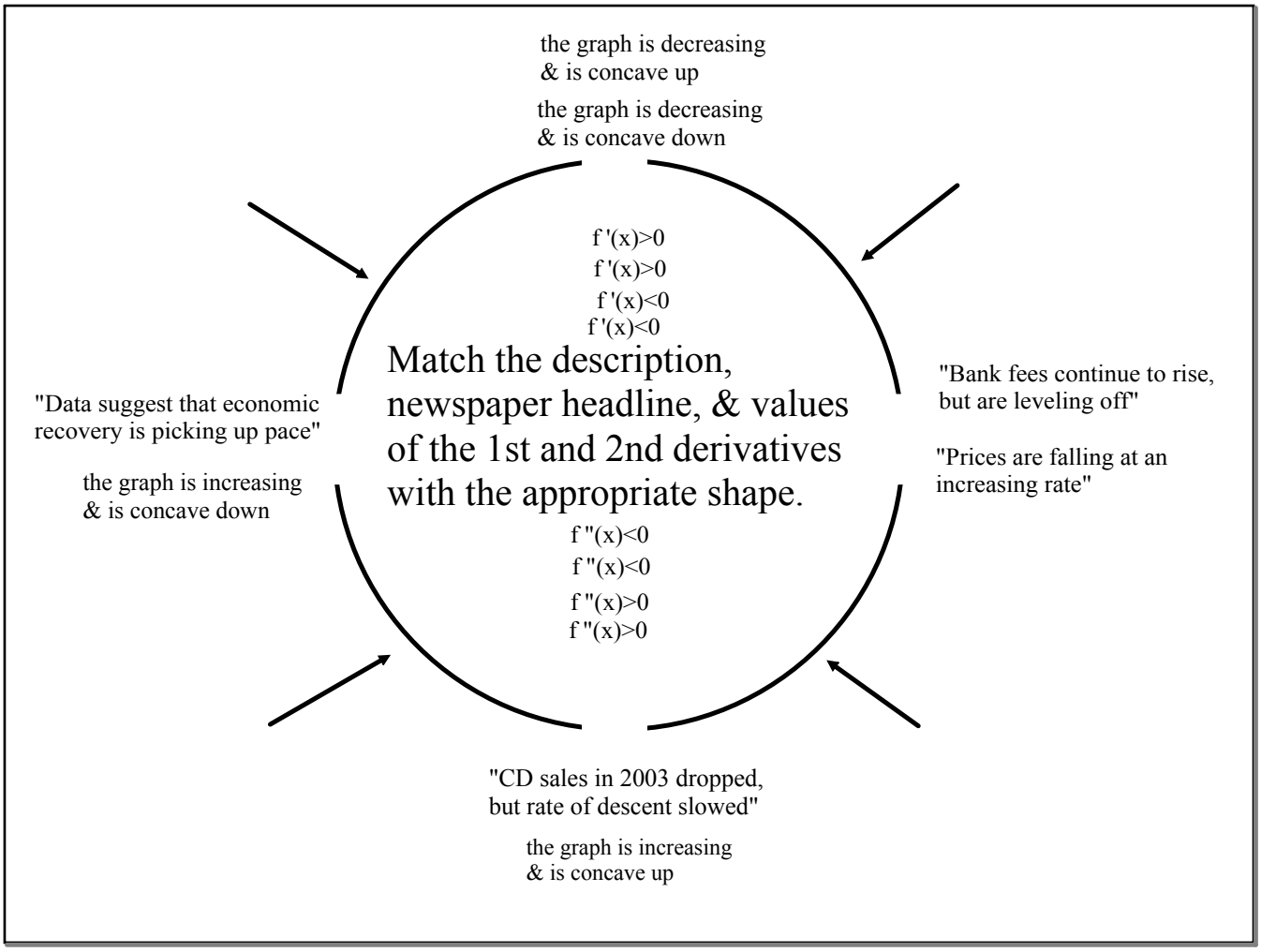
Defn. A point of inflection is a point where a graph changes from concave up to concave down or vice versa.

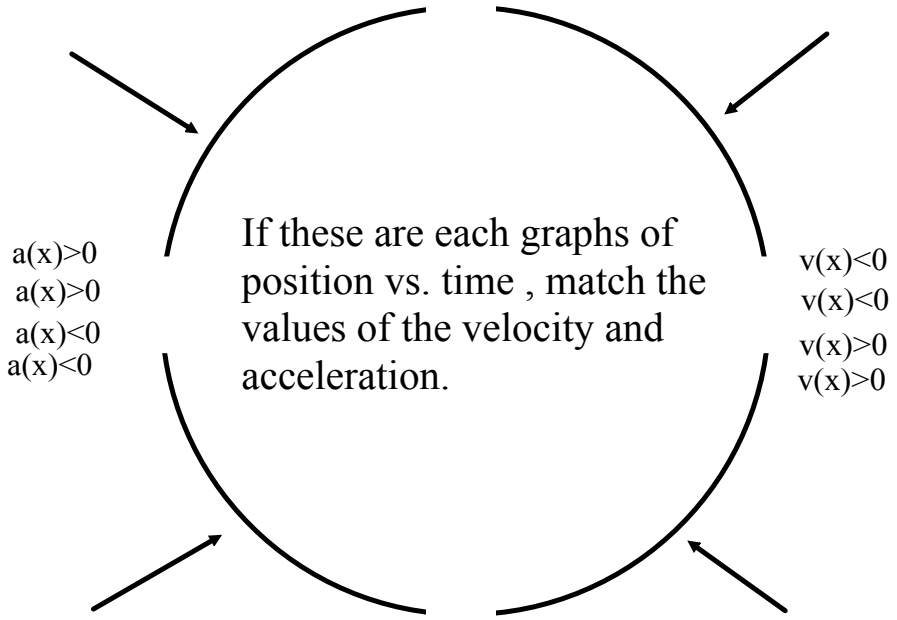
Thm. Test for Concavity

Let f be a function whose second derivative exists on an interval I, then

1. if f "(x)>0, then f is concave up.
2. if f "(x)<0, then f is concave down.

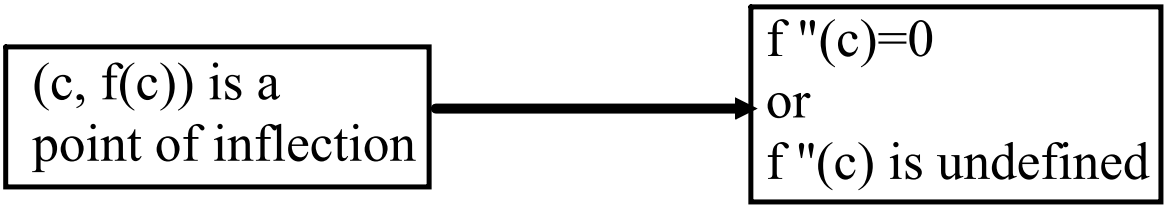



the graph is decreasing
& is concave up
the graph is decreasing
& is concave down
f '(x)>0
f '(x)>0
f '(x)<0
f '(x)<0
Match the description, newspaper headline, & values of the 1st and 2nd derivatives with the appropriate shape.
f "(x)<0
f "(x)<0
f "(x)>0
f "(x)>0
"Data suggest that economic recovery is picking up pace"
the graph is increasing
& is concave down
"Bank fees continue to rise, but are leveling off"
"Prices are falling at an increasing rate"
"CD sales in 2003 dropped, but rate of descent slowed"
the graph is increasing
& is concave up

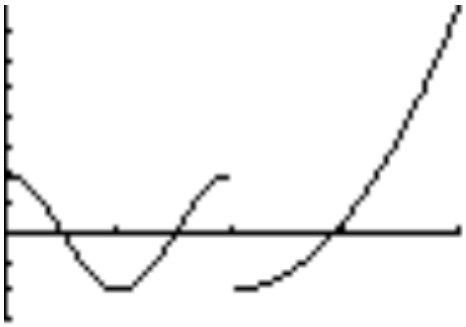


When is the speed of the object increasing or decreasing?

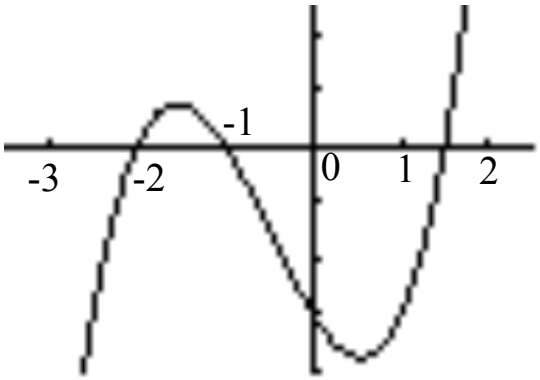
Thm.



ex. Function f(x) is graphed below and is defined on [0,4]. Estimate the intervals on which f '(x) is positive or negative and on which intervals f "(x) is positive or negative.



ex. For the graph shown, at which integer value of x is it true that both f '(x)>0 and f "(x)>0?



ex. Determine the concavity and identify any points of inflection of

$$f(x) = \frac{2}{x} + \sqrt{x}$$

ex. Find the x coordinates of any points of inflection for the graph of

$$f(x) = 3x^5 - 5x^4$$

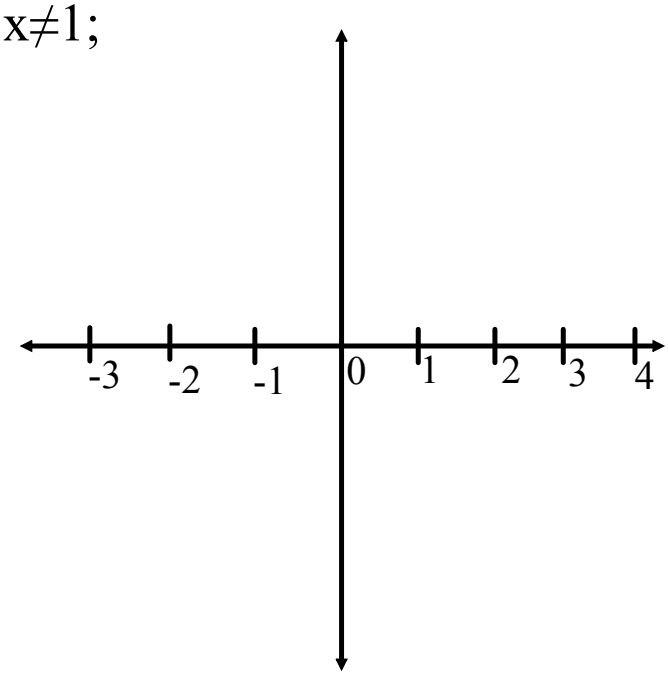
ex. Sketch the graph of a cont's function which satisfies all the following conditions:

$f'(x)<0$ for all real numbers $x \neq 1$;

$f'(1)$ does not exist;

$f''(x)<0$ for all $x<1$; and

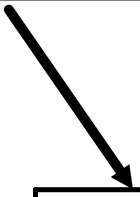
$f''(x)>0$ for all $x>1$



Thm. First Derivative Test (FDT)

f is continuous on I
f is differentiable on I, except possibly at c
c is a critical number
f "(c) is +
is -

if f " (c)=0, the SDT fails.



f(c) is a relative min
max

Review: First Derivative Test

f ' >0 on (_,c) | f '(c)=0 | f ' <0 on (c,_) | f (c) is a max

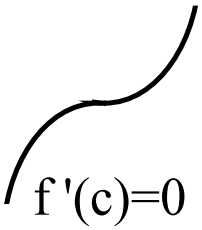
---

f ' <0 on (_,c) | f '(c)=0 | f ' >0 on (c,_) | f (c) is a min

New: Second Derivative Test

f '(c)=0 | f "(c) <0 | f (c) is a max

f '(c)=0 | f "(c) >0 | f (c) is a min



f '(c)=0 | f "(c) =0 | No conclusion
Second Derivative Test fails

Point of Inflection Test

f "(c) is 0 or undef 'd
f(c) is defined
f " changes sign at x=c

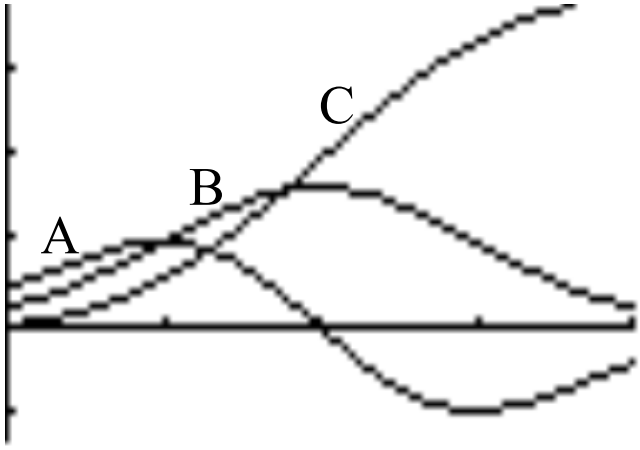
→ (c,f(c)) is a Point of Inflection

ex. A function f is cont's on [-3,3] and its first and second derivatives are as follows:

x	(-3,-1)	-1	(-1,0)	0	(0,1)	1	(1,3)
$f'(x)$	Positive	0	Negative	Negative	Negative	0	Negative
$f''(x)$	Negative	Negative	Negative	0	Positive	0	Negative

At what x values does f have...
a. relative minima? Justify.
b. relative maxima? Justify.
c. points of inflection? Justify.

ex. Which function is f, which is f ', and which is f "?



Graph a function with the following properties:

$f(2)=f(4)=0$.

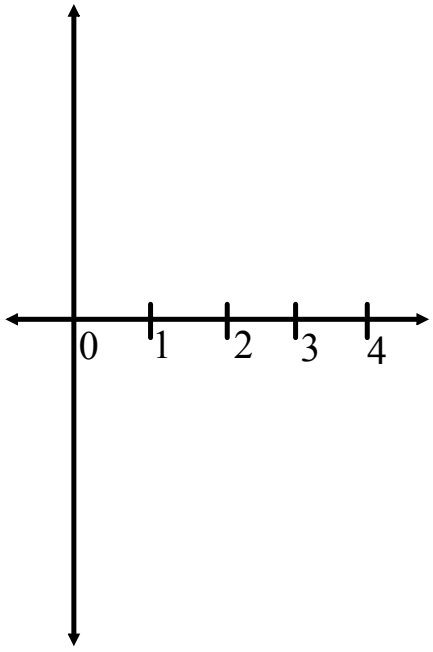
$f(3)$ is defined.

$f'(x)<0$ for $x<3$.

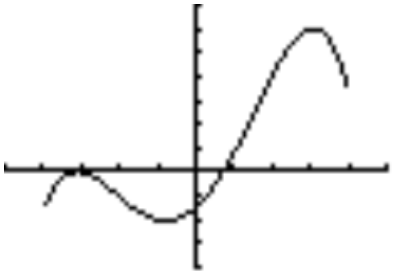
$f'(3)$ is undefined.

$f'(x)>0$ for $x>3$.

$f''(x)<0$ for $x\neq 3$.

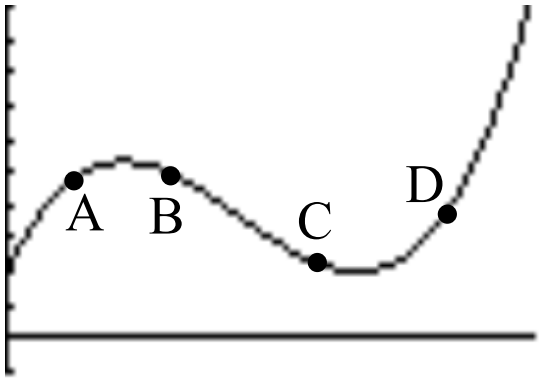


ex. The graph of f '(x) is shown.
a. Suppose f(3)=1. Find the equation of the line tangent to f at (3,1).
b. Where does f have a local minimum? Justify.
c. Estimate f "(3).
d. Where does f have an inflection point? Justify.



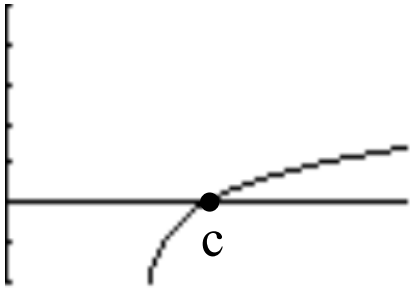
ex. At which point(s) is the first derivative of f positive?

At which point(s) is the second derivative of f positive?



ex. Arrange these in order from least to greatest:

f(c) f '(c) f ''(c)



ex. The derivative of a function f is given by

$$f'(x) = (x-3)^2(x+2)$$

Where does f have points of inflection?

ex. The derivative of a function f is given by

$$f(x) = x^4 - 4x^3 + 6x^2$$

Where does f have points of inflection?