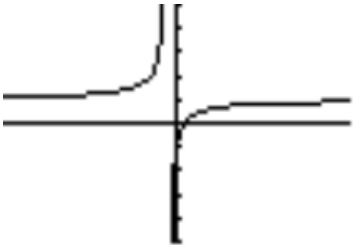
Do now as a warm-up:

Assuming the y axis is scaled in an expected way, what is the equation of any horizontal asymptotes for this graph?



Write one or more mathematical equations using limit notation that explain(s) the end behavior of this function.

3.5 Limits at Infinity

Suppose $f(x) = \dfrac{x^7 + 5x - 14}{3x - 6x^7 - 2}$.

What is $\lim\limits_{x \to \infty} \dfrac{x^7 + 5x - 14}{3x - 6x^7 - 2}$?

The math analysis or precalculus **rule**:
If numerator degree = denominator degree, then a horizontal asymptote occurs at
y= (the ratio of the coefficients of the highest powers).

Ah, but what about the calculus **methods**?

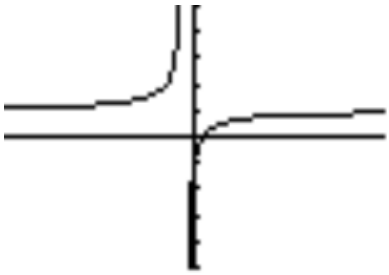
Defn. Limits at Infinity

Let $L \in R$

1. The statement

$$\lim_{x \to \infty} f(x) = L$$

means that for each $\epsilon > 0$, there exists an $m > 0$ such that $|f(x) - L| < \epsilon$ whenever $x > m$.



2. The statement

$$\lim_{x \to -\infty} f(x) = L$$

means that for each $\epsilon > 0$, there exists an $n < 0$ such that $|f(x) - L| < \epsilon$ whenever $x < n$.

Defn. Horizontal Asymptote

The line y=L is a horizontal asymptote of the graph of y if

$\lim_{x\to\infty} f(x)=L$ or $\lim_{x\to-\infty} f(x)=L$

Thm.

r>0 and c is a constant → $\lim_{x\to\infty} \frac{c}{x^r}=0$

ex. $\lim_{x\to\infty} \frac{x^7 + 5x - 14}{3x - 6x^7 - 2}$

ex. Find any horizontal asymptote for $f(x)= \frac{3x^2-x}{2x^2+1}$

ex. $\lim_{x \to \infty} \sin x$

$$\lim_{x \to \infty} \frac{2x - \sin x}{x}$$

ex. $\lim_{x \to -\infty} \frac{2x}{\sqrt{x^2 - x}}$

Caution! Any time we look for $\lim_{x \to -\infty}$, be sure to check the sign of the answer.

ex. Find all asymptotes for $y = 2 - \frac{3}{x^2}$