3.6 A summary of Curve Sketching

1. Use f to find
a. x or y intercepts
b. locations where f is undefined
c. any vertical asymptotes
2. Use f ' to find
a. intervals where increasing/decreasing
b. critical values
c. extrema
3. Use f '' to find
a. intervals where CU or CD
b. possible POI
c. extrema
4. Check limits at ± infinity for
a. horizontal asymptotes
b. other end behavior
5. Sketch a graph to match the details found.

ex. $f(x)=\frac{2(x^2-9)}{x^2-4}$

f

$0=\cancel{2}(x^2-9)$

$x=3,\ -3$ x int's

$f(0)=\frac{2(-9)}{-4}=\frac{9}{2}$ y int

$x^2-4=0$

$x=\pm 2$ VA's

since bottom is 0 but top is not.

$$f'(x)=\frac{2(2x)(x^2-4)-2(x^2-9)(2x)}{(x^2-4)^2}$$

$20x$

CV's $x=\pm 2,\ 0$

$0=\cancel{4x^3}-16x-\cancel{4x^3}+36x$

$0=20x$

$0=x$

	-2		0		2	
f'	$-$		$-$	$+$		$+$
f	decr.		decr.	incr.		incr.

min at $x=0$

min is $f(0)=\frac{9}{2}$

$$f'(x)=\frac{20x}{(x^2-4)^2}$$

$$f''(x)=\frac{20(x^2-4)^{\cancel{2}}-20x(2)\cancel{(x^2-4)}(2x)}{(x^2-4)^{\cancel{4}\,3}}$$

$$=\frac{20x^2-80-80x^2}{(x^2-4)^3}$$

$$=\frac{-80-60x^2}{(x^2-4)^3}$$

undefined at $x=\pm 2$

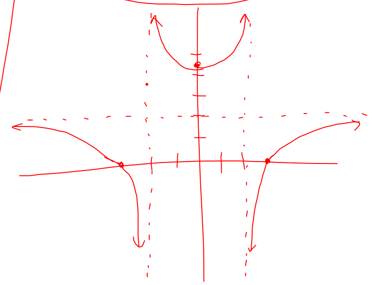
$-80-60x^2=0$

$-80=60x^2$

$\varnothing$

		-2		2	
f''	$-$		$+$		$-$
f	CD		CU		CD

not VA's POI



$$\lim_{x\to\pm\infty}\frac{2(x^2-9)}{(x^2-4)}=2$$

HA at $y=2$

ex. $f(x)= x^4-12x^3+48x^2-64x$

$f(0) = 0^4-12\cdot 0^3+48\cdot 0^2-64\cdot 0 = 0$ y-int

$0 = x^4-12x^3+48x^2-64x$

$0 = x(x^3-12x^2+48x-64)$

$64 \rightarrow p = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64$

$1 \rightarrow q = \pm 1$

$$\begin{array}{r|rrrr} 4 & 1 & -12 & 48 & -64 \\ & & 4 & -32 & 64 \\ \hline & 1 & -8 & 16 & 0 \end{array}$$

$x^2-8x+16=0$

$x(x-4)$ $(x-4)(x-4)=0$

$x=0,\ x=4,\ x=4,\ x=4$ ← x intercepts

always defined → no VA's

$f'(x) = 4x^3-36x^2+96x-64$

$\frac{0}{4} = \frac{4x^3}{4} \frac{-36x^2}{4} \frac{+96x}{4} \frac{-64}{4}$

$0 = x^3-9x^2+24x-16$

$16 \rightarrow p = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$1 \rightarrow q = \pm 1$

$$\begin{array}{r|rrrr} 1 & 1 & -9 & 24 & -16 \\ & & 1 & -8 & 16 \\ \hline & 1 & -8 & 16 & 0 \end{array}$$

$x^2-8x+16=0$

$(x-1)$ $(x-4)(x-4)=0$

$x=1,\ x=4$ ← CV's

1, 4

f′: − + +

f: dec. inc. inc.

min at $x=1$

min is $f(1) = 1-12+48-64$

$= -27$

$f''(x) = 12x^2-72x+96$

$\frac{0}{12} = \frac{12x^2}{12} \frac{-72x}{12} \frac{+96}{12}$

$0 = x^2-6x+8$

$0 = (x-4)(x-2)$

$x = 4, 2$

2, 4

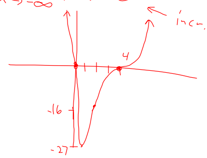
f″: + − +

f: CU CD CU

$(2,-16)$ & $(4,0)$ are P.o.i.

$\lim_{x\to\infty} f(x) = \infty$ ← incr. to the right.

$\lim_{x\to-\infty} f(x) = \infty$ ← incr. to the left.



ex. $f(x) = \sin x - \cos x$ on $[0, 2\pi]$

$f(0) = \sin 0 - \cos 0 = 0 - 1 = -1$ y int

$0 = \sin x - \cos x$

$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$ or $\tan x = 1$

$x = \frac{\pi}{4}$ & $\frac{5\pi}{4}$ x int

always defined → no VA's

$f'(x) = \cos x + \sin x$

$0 = \cos x + \sin x$

$\sin x = -\cos x$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$ CV's

$0 \quad \frac{3\pi}{4} \quad \frac{7\pi}{4} \quad 2\pi$

f': + | − | +

f: inc. | dec. | inc.

max at $\frac{3\pi}{4}$

max is $\sqrt{2}$

min at $\frac{7\pi}{4}$

min is $-\sqrt{2}$

$f''(x) = -\sin x + \cos x$

$0 = -\sin x + \cos x$

$\sin x = \cos x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$0 \quad \frac{\pi}{4} \quad \frac{5\pi}{4} \quad 2\pi$

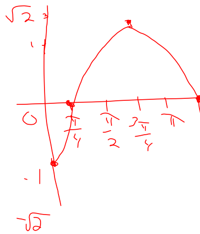
f'': + | − | +

f: CU | CD | CU

$\left(\frac{\pi}{4}, 0\right)$ p.o.i.

$\left(\frac{5\pi}{4}, 0\right)$ p.o.i.

~~lim x→∞~~ $[0, 2\pi]$



should appear smooth