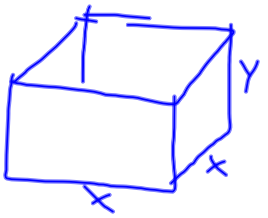
3.7 Optimization Problems

ex. A manufacturer wants to design an open (lidless) box having a square base and a surface area of 108 sq. in.. What dimensions will produce a maximum volume?



$SA = \boxed{108 = x^2 + 4xy}$

$\frac{108 - x^2}{4x} = \frac{4xy}{4\cancel{x}}$

$\frac{108 - x^2}{4x} = y$

$V = x^2 y$

$V = \frac{x}{1}^{\cancel{2}}\left(\frac{108 - x^2}{4\cancel{x}}\right)$

$V = \frac{108x - x^3}{4}$

$V = 27x - \frac{1}{4}x^3$

$V' = 27 - \frac{3}{4}x^2$

$0 = 27 - \frac{3}{4}x^2$

$\frac{4}{3} \cdot \frac{3}{4}x^2 = 27 \cdot \frac{4}{3}$

$x^2 = 36$

$x = \pm 6$ use $x = 6$

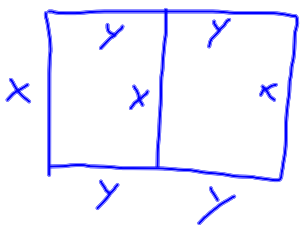
V' changes + to − at $x = 6$, so this gives a max. vol.

$y = \frac{108 - 6^2}{4 \cdot 6} = \frac{72}{24}$

$= 3$

$6 \times 6 \times 3$

ex. A farmer has 400 feet of fence and wants to make 2 congruent rectangular hog lots adjacent to one another. What dimensions will yield a maximum area for the animals?



$400 = 3x + 4y \rightarrow 4y = 400 - 3x$

$A = 2xy$

$y = 100 - \frac{3}{4}x$

$A = 2x\left(100 - \frac{3}{4}x\right)$

$A = 200x - \frac{3}{2}x^2$

$A' = 200 - 3x$

$0 = 200 - 3x$

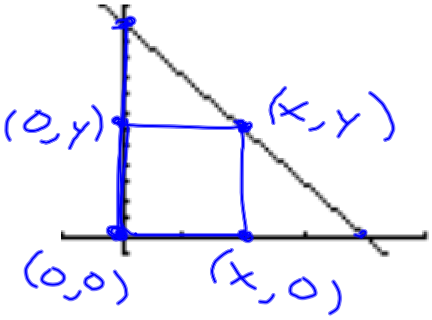
$\frac{200}{3} = x \longrightarrow y = 100 - \frac{3}{4}\left(\frac{200}{3}\right)$

$y = 50$

each pig lot is

$\frac{200}{3}$ ft by 50 ft

ex. A rectangle has opposite corners at the origin and on the line y=-3x+12. What dimensions will yield a maximum area?



$A = xy = x(-3x+12) = -3x^2+12x$

$A' = -6x+12$

$0 = -6x+12$

$2 = x$

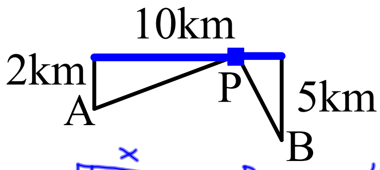
$y = -3(2)+12 = 6$

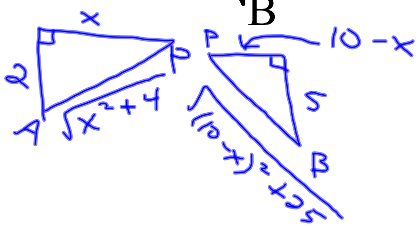
2 by 6, which is a max area because

$A' > 0$ for any $x < 2$

& $A' < 0$ for any $x > 2$

ex. Two towns near the south bank of a straight stretch of river plan to build and share a pumping station so that each has a direct pipeline to the station. Where should the pumping station be located to minimize the amount of pipeline constructed?





$$D = \sqrt{x^2+4} + \sqrt{(10-x)^2+25}$$

$$D = (x^2+4)^{\frac{1}{2}} + (x^2-20x+125)^{\frac{1}{2}}$$

$$D' = \cancel{\tfrac{1}{2}}(x^2+4)^{-\frac{1}{2}}(\cancel{2}x) + \cancel{\tfrac{1}{2}}(x^2-20x+125)^{-\frac{1}{2}}(\cancel{2}x - \cancel{20})$$

1x − 10

$$D' = \frac{x}{\sqrt{x^2+4}} + \frac{x-10}{\sqrt{x^2-20x+125}}$$

$D' = 0$ when $x \approx 2.857$

$$0 = \frac{x}{\sqrt{x^2+4}} + \frac{x-10}{\sqrt{x^2-20x+125}}$$

$$\frac{-x}{\sqrt{x^2+4}} = \frac{x-10}{\sqrt{x^2-20x+125}}$$

$$-x\sqrt{x^2-20x+125} = (x-10)\sqrt{x^2+4}$$

$$x^2(x^2-20x+125) = (x^2-20x+100)(x^2+4)$$

$$\cancel{x^4} - \cancel{20x^3} + 125x^2 = \cancel{x^4} - \cancel{20x^3} + 100x^2 + 4x^2 - 80x + 400$$

$$125x^2 = 104x^2 - 80x + 400$$

$$21x^2 + 80x - 400 = 0$$

$x \approx 2.857142857$, ~~$x \approx -6\frac{2}{3}$~~ 2 critical #'s

$2\frac{6}{7}$