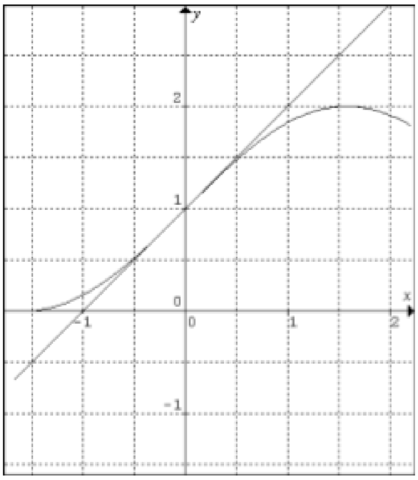
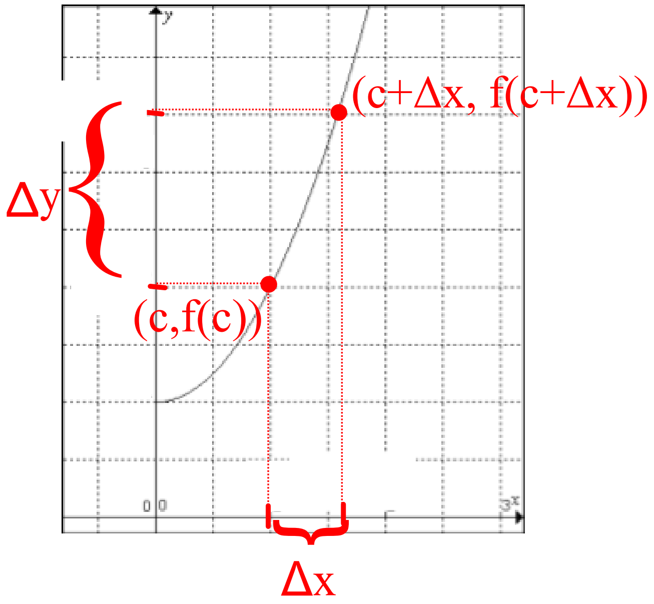
3.9 Differentials (linear approximations)

Use the tangent line to f(x)=1+sinx at x=0, y=x+1, to approximate f(0.5).



The difference between two y values on the graph of a function is Δy=f(c+Δx)-f(c).



That difference is close to f '(c)Δx.

Δx is often written dx. dx is called the differential of x.

Then f '(c)Δx can be written f '(c)dx and is called the differential of y.

Defn. Let $y=f(x)$ be differentiable, then the differential of x, written dx, is any non-zero number and the differential of y, written dy, is $dy=f'(x)dx$.

Note: $dy \approx \Delta y$

$dx = \Delta x$

Compare Δy to dy for the function $y=x^2$
when x=1 and dx=.01.

$dy=f'(c)dx$

$\Delta y=f(c+\Delta x)-f(c)$

What is the % error?

Error terms used in our book:

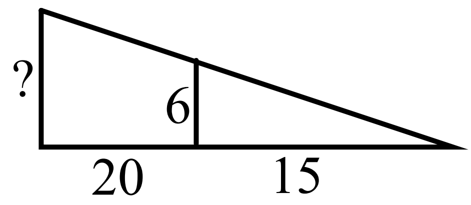
$$\underbrace{\Delta y}_{\text{propagated error}} = \overbrace{f(x+\underbrace{\Delta x}_{\text{measurement error}})}^{\text{exact value}} - \overbrace{f(x)}^{\text{measured value}}$$

ex. The radius of a sphere is measured to be 0.7 inch. The measurement could be off by as much as ± 0.01 inch. Estimate the error in calculating the volume of the sphere.

ex. Find dy if y= $3x^2 - 4$

ex. Find dy if y= $\tan^2 x$

ex. To find the height of a lamppost, we stand a 6' pole 20' from the lamppost's base & measure the pole's shadow to be 15' long. If we believe our measurement could be off by as much as 1", what is our estimate of the lamppost's height?



ex. Find the linear approximation of the function

$$f(x) = \sqrt[3]{x}$$

at a=8 and use it to approximate the number $\sqrt[3]{7.99}$

ex. Use differentials to approximate $\sqrt{24.8}$