# 4.1 Antiderivatives and Indefinite Integrals

Defn. A function F(x) is an antiderivative of f(x) on an interval I if F '(x)=f(x) for all x in I.

ex. Find an antiderivative of f(x)=2x $^{1}$

$F(x) = \frac{\cancel{2}x^2}{\cancel{2}} + C$

ex. Find an antiderivative of f(x)=sinx

$F(x) = -\cos x + C$

ex. Find an antiderivative of f(x)= $\frac{1}{x^2} = x^{-2}$

$F(x) = \frac{x^{-1}}{-1} + C = -\frac{1}{x^1} + C$

Thm. If F is an antiderivative of f on an interval I, then G is an antiderivative of f iff G is of the form $G(x)=F(x)+C$, where C is a constant.

G is called a General Antiderivative.
C is called the Constant of Integration.

Defn. A Differential Equation in x and y is an equation that involves derivatives of y.

These all mean the same thing:

Find the general solution of a differential equation.

Find an antiderivative of a function.

Evaluate an indefinite integral.

ex. Find a general solution for the differential equation y'=3x.

$$\frac{dy}{dx} = 3x \rightarrow \int dy = \int 3x^1\,dx$$

$$y = \frac{3x^2}{2} + C$$

ex. Find the particular solution for the differential equation y'=3x if x=1 when y=2/3.

$$\frac{2}{3} = \frac{3(1)^2}{2} + C$$

$$\frac{2}{3} = \frac{3}{2} + C$$

$$\frac{2}{3} - \frac{3}{2} = C$$

$$\frac{4}{6} - \frac{9}{6} = C$$

$$C = -\frac{5}{6}$$

so, $y = \frac{3x^2}{2} - \frac{5}{6}$

Notation

$$\frac{dy}{dx} = f\,'(x)$$

$$dy = f\,'(x)\,dx$$

The constant of integration.

$$\int dy = \int \underbrace{f\,'(x)}_{\text{integrand}}\, dx = \overbrace{F(x)}^{\text{The antiderivative}} + C$$

This shows the variable of integration.

This is an antiderivative symbol, also called an integral symbol. It's an elongated S.

Defn. The operation of finding antiderivatives is called <u>indefinite integration</u>.

Basic Integration Rules

1. $\int f'(x)dx = f(x) + C$
2. $\frac{d}{dx}[\int f(x)dx] = f(x)$
3. $\int 0 \, dx = C$

So, integration and differentiation are inverse operations (just like + and -, x and ÷, or squaring and square rooting).

4. For some constant k, $\int k \, dx = kx + C$
5. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$
6. $\int \sin x \, dx = -\cos x + C$
7. $\int \cos x \, dx = \sin x + C$
8. $\int \sec^2 x \, dx = \tan x + C$
9. $\int \csc^2 x \, dx = -\cot x + C$
10. $\int \sec x \tan x \, dx = \sec x + C$
11. $\int \csc x \cot x \, dx = -\csc x + C$

ex. $\int (x^2 + 2)dx = \frac{x^3}{3} + 2x + C$

ex. $\int 3\sqrt{x}\, dx = \int 3x^{\frac{1}{2}} dx = \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{2}{3} \cdot 3x^{\frac{3}{2}} + C = 2x^{\frac{3}{2}} + C$

$\cancel{x}(x^3 + x)$

ex. $\int \frac{x^4+x^2}{\cancel{x}}\, dx = \int x^3 + x \, dx$

$= \frac{x^4}{4} + \frac{x^2}{2} + C$

ex. If dy/dx = 2x-1 what is the general solution of this differential equation?

$$\int dy = \int (2x-1)\,dx$$

$$y = x^2 - x + C$$

What is the particular solution of this differential equation for x=1 and y=1?

$$1 = 1^2 - 1 + C \qquad y = x^2 - x + 1$$

$$1 = C$$

ex. Let f(x) be the derivative of $x \bullet \sin\left(x^2\right)$
Find $\int f(x)dx$

$$f(x) = \frac{d}{dx}\left[x \cdot \sin(x^2)\right]$$

$$\int f(x)\,dx = \cancel{\int \frac{d}{dx}}\left[x \sin x^2\right] dx$$

$$= x \sin x^2 + C$$

Mom $x \sin x^2$

deriv. Kid $f$

ex. Suppose that the acceleration of a particle is $a(t)=\cancel{x}^{-3/2}$ $t$
Find its position function x(t) if v(4)=2 and x(0)=0.

$$v(t)=\int a(t)\,dt=\int t^{-3/2}\,dt$$

$$v(t)=\frac{t^{-\frac{1}{2}}}{-\frac{1}{2}}+C_1$$

$$v(t)=-2t^{-1/2}+C_1$$

$$2=-2\cdot 4^{-1/2}+C_1$$

$$2=\frac{-2}{2}+C_1$$

$$3=C_1$$

$$\int v(t)\,dt=\int -2t^{-\frac{1}{2}}+3\,dt$$

$$x(t)=\frac{-2t^{\frac{1}{2}}}{\frac{1}{2}}+3t+C_2$$

$$x(t)=-4t^{1/2}+3t+C_2$$

$$0=0+0+C_2$$

$$x(t)=-4t^{1/2}+3t$$