Do now as a warm-up:

A tank is being filled with water using an old pump that slows down as it runs. The table below gives the rate at which the pump pumps at 10 minute intervals. If the tank is initially empty, estimate how much water is in the tank after 90 minutes.

Elapsed time (minutes)	0	10	20	30	40	50	60	70	80	90
Rate (gallons/minute)	42	40	38	35	35	32	28	20	19	10

The speed of an airplane in miles per hour is given at half-hour intervals in the table below. Approximately, how far does the airplane travel in the 3 hours given in the table? Can we say how far it is from the airport?

Elapsed time (minutes)	0	30	60	90	120	150	180
Speed (miles/hour)	375	390	400	390	385	350	345

teacher notes:
left, right, upper, lower, trapezoidal sums
midpoint sum
units or dimensions

4.2 Area

Sigma Notation

The sum of the first n terms a_1, a_2, a_3, ..., a_n is written as

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \ldots + a_n$$

where i is the index of summation, a_i is the ith term, and the upper and lower bounds of the sum are 1 and n.

ex. $\sum_{i=1}^{6} i$

ex. $\sum_{i=2}^{7} i^2$

ex. $\sum_{i=1}^{4} 5$

Thm. Summation Formulas when i begins at 1

1. $\sum_{i=1}^{n} c = cn$

2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4. $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

ex. $\sum_{i=1}^{50} i^2$

ex. Write the sigma notation for $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots+\frac{1}{128}$

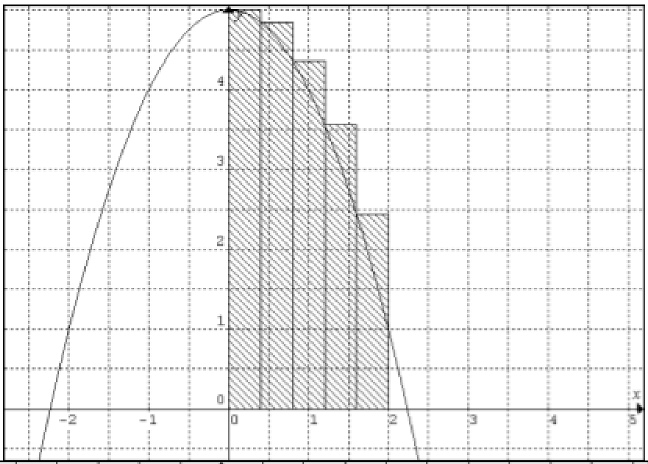
ex. Write the sigma notation for

$$[2\left(\frac{1}{8}\right)+3]+[2\left(\frac{2}{8}\right)+3]+[2\left(\frac{3}{8}\right)+3]+\ldots+[2\left(\frac{8}{8}\right)+3]$$

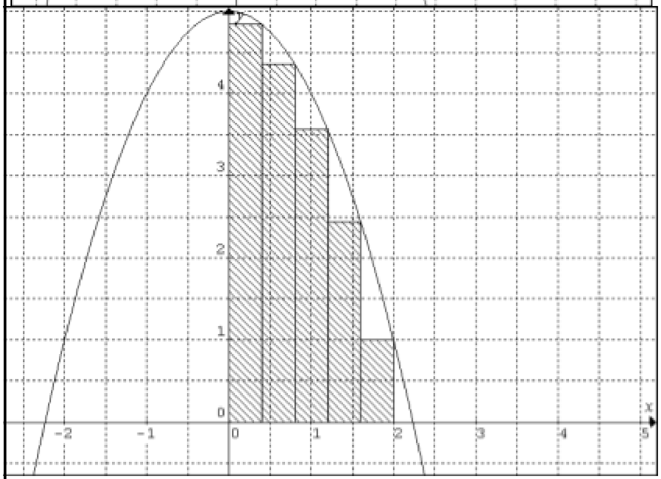
Let S(n) denote the sum of the first n terms of an infinite sum.

ex. Suppose that $S(n)= \frac{4}{3n^3}(2n^3+3n^2+n)$

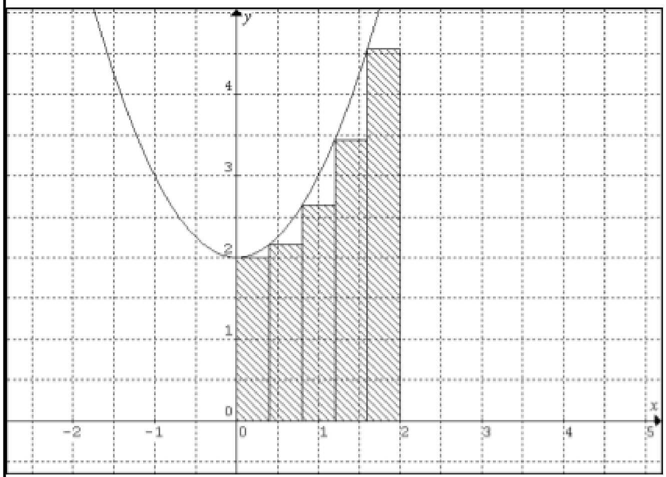
Find $\lim_{n\to\infty} S(n)$



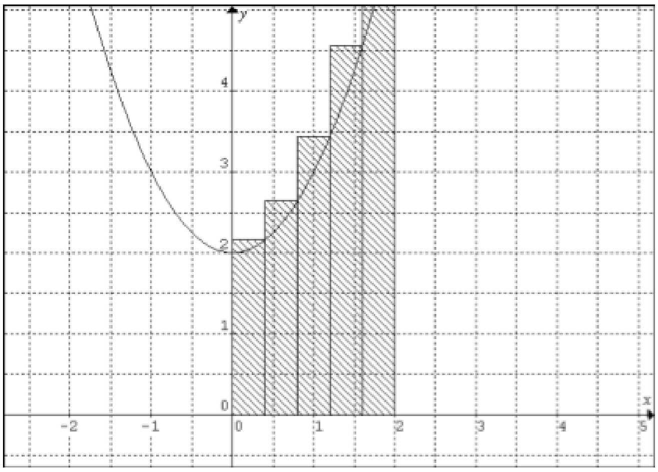
This is an upper or left approximating sum.



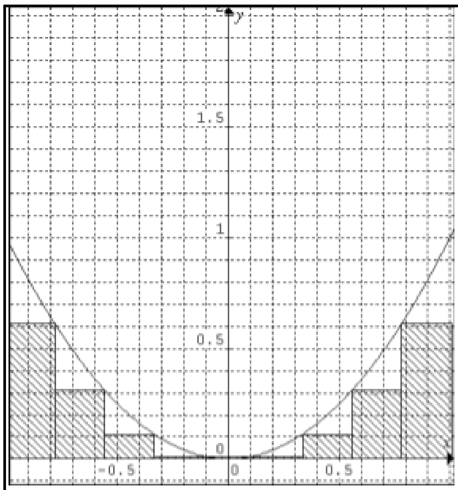
This is a lower or right approximating sum.



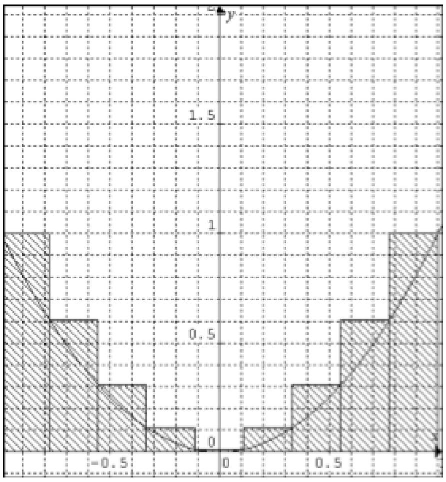
This is a lower or left approximating sum.



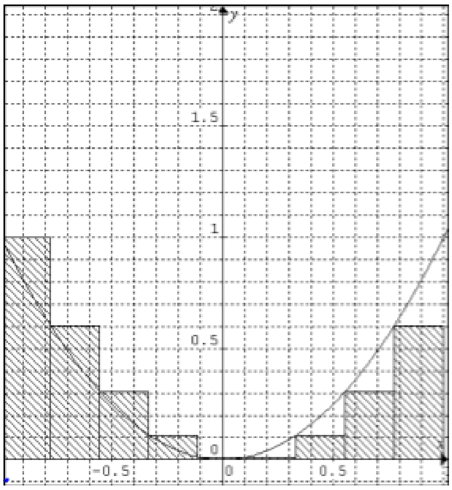
This is an upper or right approximating sum.



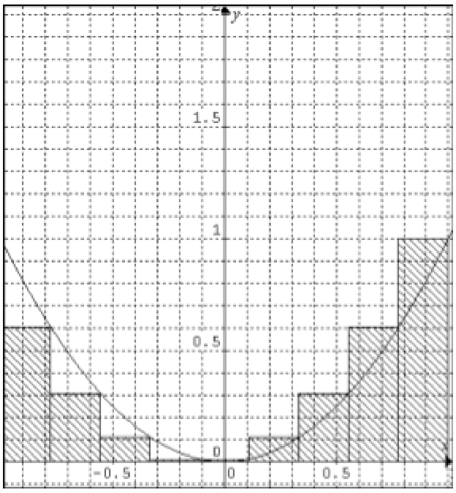
This is a lower approximating sum.



This is an upper approximating sum.



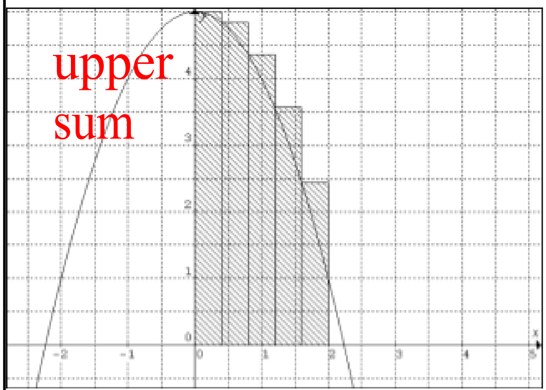
This is a left approximating sum.

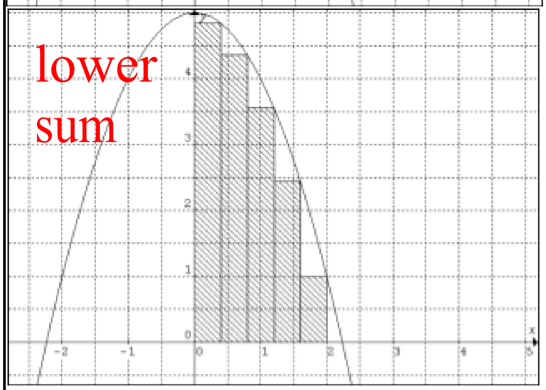


This is a right approximating sum.

Teacher note: visit APCD for left, right, midpoint, and trapezoid sums

ex. Find the upper and lower approximating sums to approximate the area under f(x)= $-x^2+5$
between x=0 and x=2 using 5 rectangles.





ex. Average these for the trapezoidal approximation.

Thm. Limit of the lower, upper, left, and right sums

f is continuous and non-negative on [a,b]	→	As $n \to \infty$, the lower, upper, left, and right sum limits exist and are all equal.

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(m_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} f(M_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} S(n)$$

where $\Delta x = \dfrac{b-a}{n}$

and $f(m_i)$ is the minimum, and $f(M_i)$ is the maximum.

Defn. Area of a region

Let f be continuous and non-negative on the interval [a,b], then the area bounded by x=a, x=b, the graph of f, and the x axis is

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

where $\Delta x = \frac{b-a}{n}$ is the width of a rectangle

and $f(x_i)$ is the height at any x value in the i^{th} interval.