4.4 The fundamental Theorem of Calculus

Thm. The fundamental Theorem of Calculus

If a function is continuous on a closed interval [a,b] and F is the antiderivative of f(x) on [a,b], then

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

ex. $\int_{-2}^{3} x \, dx$

ex. $\int_{-1}^{1} x^3 \, dx$

ex. $\int_1^4 \left(3\sqrt{x}+x\right)\ dx$

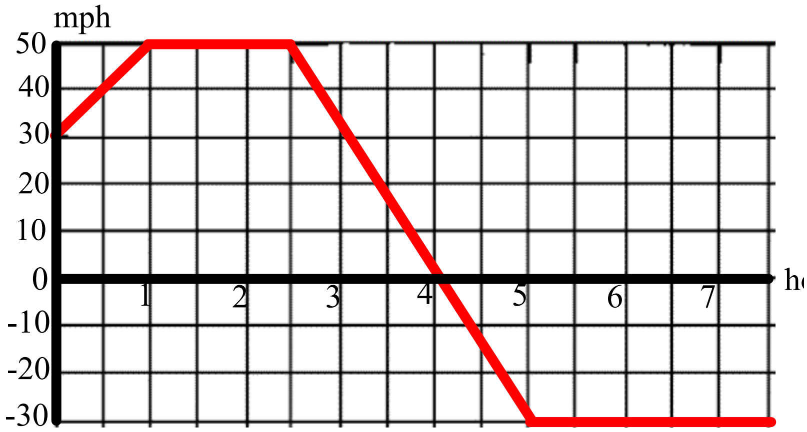
ex. $\int_0^2 \left|2x-1\right|\ dx$

ex. Find the area bounded by the function and the x axis.

f(x)= $4-x^2$

f(x)=sinx on [0,2pi]

The graph below gives the velocity of a car traveling along a straight highway. At t=0, the car is traveling toward Boston, 180 miles away.



When did the car change direction?

How many miles did the car travel between t=0 and t=3 hours?

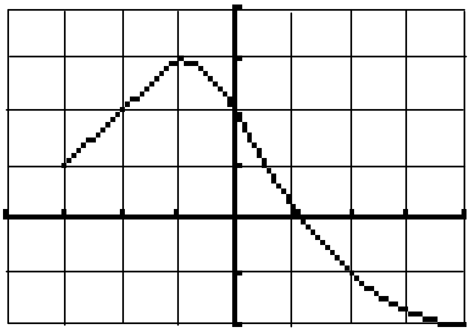
When was the car closest to Boston? Why?

When was the car farthest from Boston? Why?

Did the car reach Boston during the 7.5 hour trip?

Let $A(x) = \int_{-3}^{x} f(t)dt$

on [-3,4] where f is graphed at right.



Where is A increasing?

Explain why A has a local maximum at x=1.

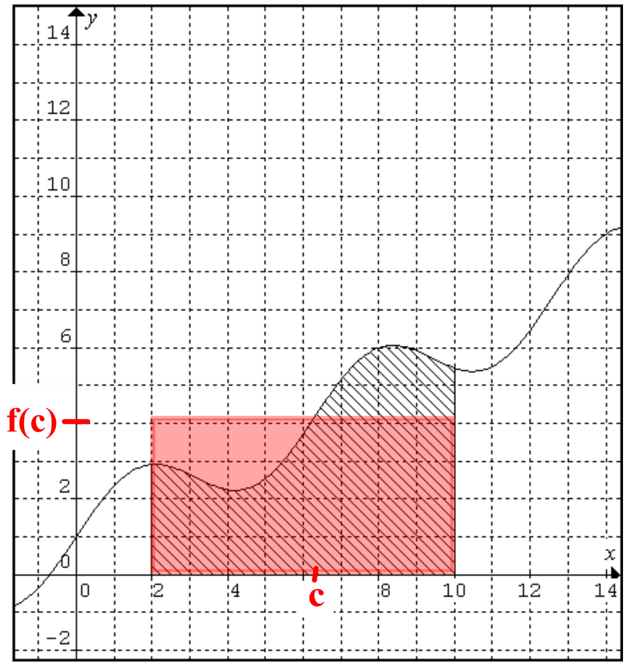
Which is larger, A(-1) or A(1). Justify your answer.

Which is larger, A(2) or A(4). Justify your answer.

Thm. The Mean Value Theorem for Integrals (MVT)

If f is continuous on a [a,b] then

$\exists c \in [a,b]$, such that $\int_a^b f(x)\ dx = f(c)(b-a)$



The area of the red box is f(c)(10-2) and that area is the same as the shaded portion under the graph of f(x) from 2 to 10.

Defn. The Average Value of a Function

If f is integrable on a [a,b] then the average value of f on [a,b] is $\frac{1}{(b-a)}\int_a^b f(x)\,dx$

This average value of f is equal to the f(c) in the MVT.

Thm. If f is continuous, then it is integrable.

ex. Find the average value of f(x)= $3x^2-2x$ on [1,4].

Thm. The Second Fundamental Theorem of Calculus

If f is continuous on an open interval I containing x, then

$$\frac{d}{dx}\int_{\mathrm{a}}^{x} f(t)\,\mathrm{dt} = f(x)$$

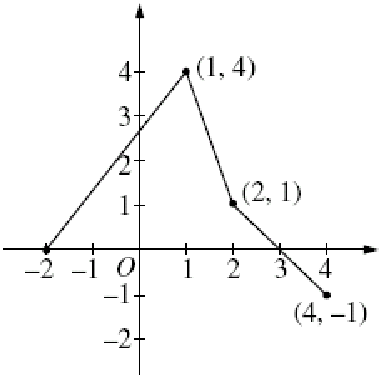
(Note: The chain rule still applies.)

ex. $\dfrac{d}{dx}\displaystyle\int_{3}^{x} \sqrt{t}\,dt$

ex. $\dfrac{d}{dx}\displaystyle\int_{3}^{x^2} \sqrt{t}\,dt$

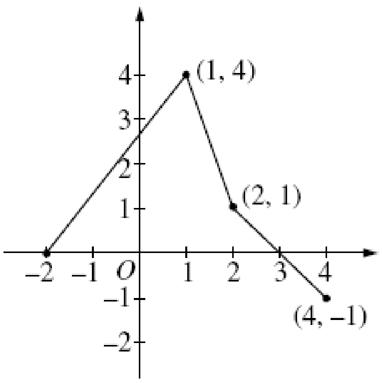
ex. $\dfrac{d}{dx}\displaystyle\int_{\sin x}^{4} \sqrt{t}\,dt$

Find the derivative of $\int_{0}^{x^{10}} \cos\sqrt{t}\, dt$



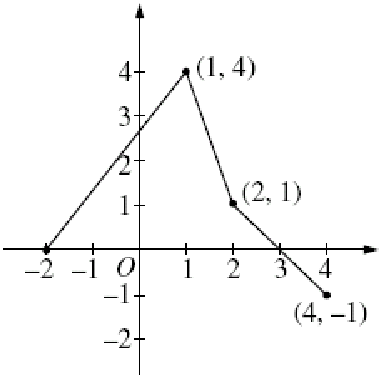
5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)dt$.

(a) Compute $g(4)$ and $g(-2)$.



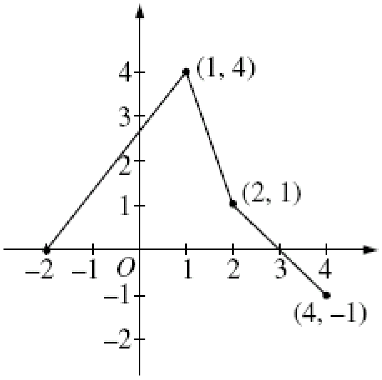
5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)dt$.

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.



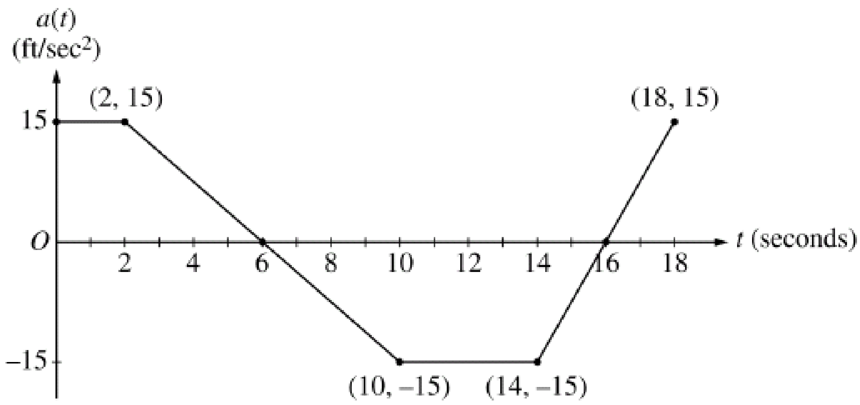
5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)dt$.

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.



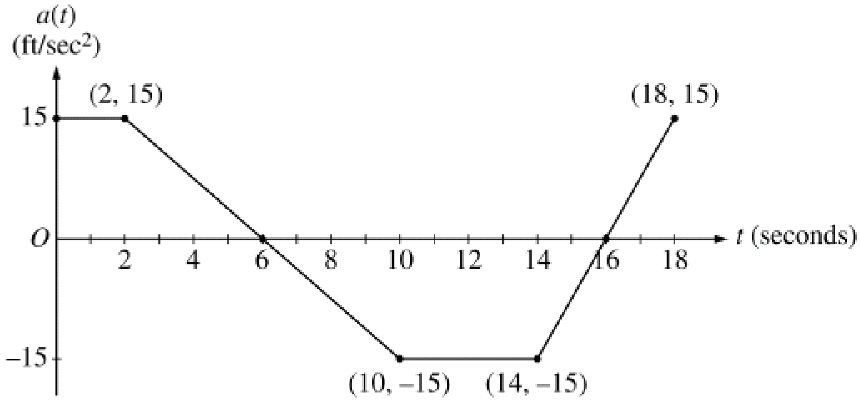
5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)\,dt$.

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g ? Justify your answer.



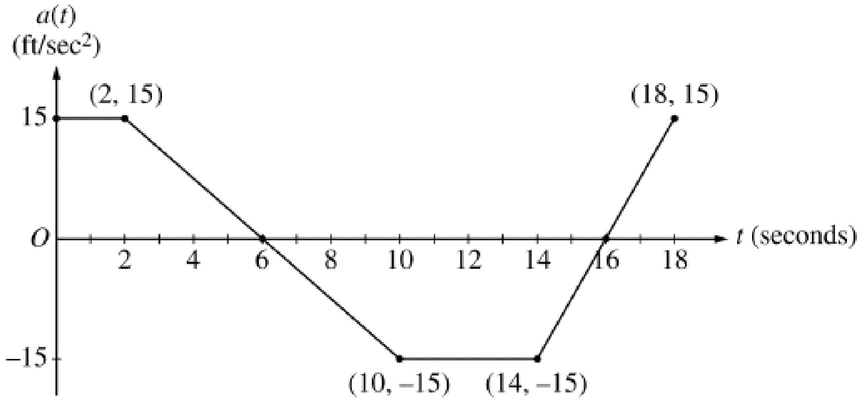
3. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?



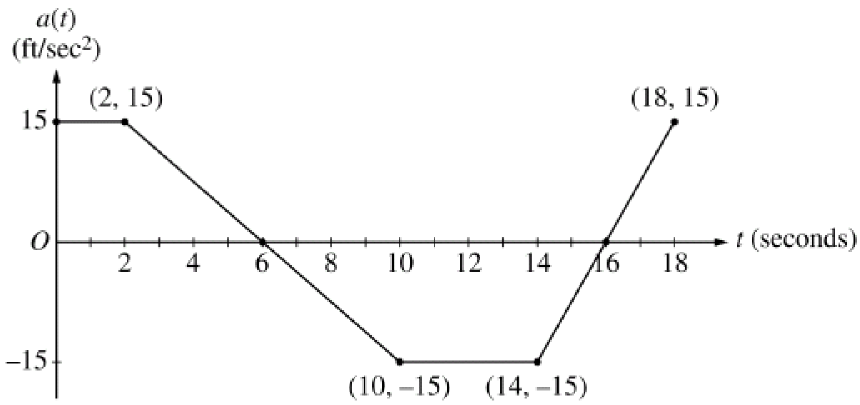
3. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?



3. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.



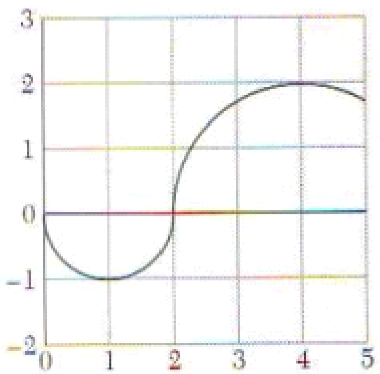
3. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

Let $g(u) = \int_1^u f(x)\,dx$ where f is the function graphed below.

(a) Evaluate $g'(4)$.
(b) Where in the interval $[0, 5]$ is g concave up? Justify your answer.
(c) How many roots does g have in the interval $[0, 5]$? Justify your answer.
(d) Rank the six numbers -1, 0, 1, $g(0)$, $g(2)$, and $g(4)$.
(e) Is the average value of g' over the interval $[0, 3]$ greater than 1? Justify your answer.

Graph of f



http://archives.math.utk.edu/visual.calculus/4/ftc.2/index.html

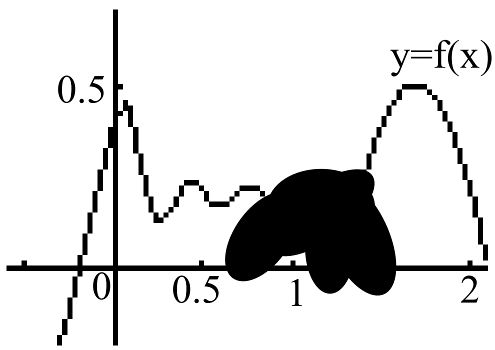
http://archives.math.utk.edu/visual.calculus/4/ftc.9/

Suppose you know that a certain function f is twice differentiable and that its graph over [0, 2] is given in the figure below. As you see, I accidentally spilled ink on a portion of the graph. Decide, if possible, whether each of the following definite integrals is positive, equal to zero, or negative.

$$\int_0^2 f''(x)dx$$

$$\int_0^2 f'(x)dx$$

$$\int_0^2 f(x)dx$$



$$\lim_{n\to\infty}\left[\left(\left(1+\frac{3}{n}(1)\right)^2+\left(1+\frac{3}{n}(2)\right)^2+\left(1+\frac{3}{n}(3)\right)^2+\ldots+\left(1+\frac{3}{n}(n)\right)^2\right)\left(\frac{3}{n}\right)\right]$$

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\Delta x(k)\right)^2\Delta x=\int_{1}^{4}x^2dx$$

or

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\Delta x(k)\right)^2\Delta x=\int_{0}^{3}(1+x)^2dx$$

Write a definite integral for each:

$$\lim_{n\to\infty}\left[\left(\frac{1}{\sqrt{n}}+\frac{\sqrt{2}}{\sqrt{n}}+\frac{\sqrt{3}}{\sqrt{n}}+\ldots+\frac{\sqrt{n}}{\sqrt{n}}\right)\frac{1}{n}\right]$$

$$\lim_{n\to\infty}\left[\left(\left(\frac{1}{n}\right)^{14}+\left(\frac{2}{n}\right)^{14}+\left(\frac{3}{n}\right)^{14}+\ldots+\left(\frac{n}{n}\right)^{14}\right)\frac{1}{n}\right]$$

$$\lim_{n\to\infty}\left[\left(\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\ldots+\frac{1}{n+n}\right)\right]$$

$$\lim_{n\to\infty}\left[\frac{1+\sqrt{2}+\sqrt{3}+\ldots+\sqrt{n}}{n^{3/2}}\right]$$