4.5 Integration by Simple Substitution

Thm. Antiderivative of a Composite Function

$$\int f(g(x))g'(x)dx = F(g(x)) + C$$

or, if v=g(x), then dv=g '(x) dx and $\int f(v)dv = F(v) + C$

ex. $\int x^2\ (3x^3 + 2)^2\ dx$

ex. $\int 5\cos(5x)dx$

If the substitution $x = \cos\theta$ is made, then

$$\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$$

is equivalent to which one(s) of the following:

$$-\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} d\theta \qquad \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} d\theta \qquad \frac{\pi}{6}$$

If the substitution $u = 1 + \sqrt{x}$ is made, then $\int_0^1 \frac{\sqrt{x}}{1+\sqrt{x}}dx$

is equivalent to which one of the following:

$2\int_1^2 \frac{u-1}{u}du$ $\qquad 2\int_1^2 \frac{(u-1)^2}{u}du$ $\qquad 2\int_0^1 \left(1-\frac{1}{u}\right)du$

$\int_1^2 \left(2u-4+\frac{2}{u}\right)du$ $\qquad 2\int_0^2 \frac{(u-1)^2}{u}du$

If the substitution $x = 2\sin y$ is made, then $\int_0^2 x^3\sqrt{4 - x^2}\,dx$

can be re-written as...

ex. $\int x \sqrt{(x^2+1)}\, dx$

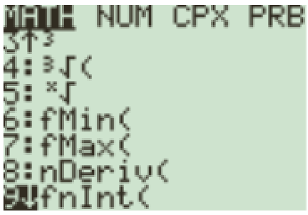
ex. $\int \frac{-x}{(x^2-4)^3}\, dx$

by substitution, by change limits, by calculator

ex. $\int_0^2 \frac{x}{\sqrt{1+2x^2}}dx$

Definite Integrals on the calculator

Press the MATH button
Choose 9:fnInt(
and enter the following:



fnInt(function,variable, lower limit, upper limit)

Thm. The General Power Rule for Integration

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C$$

for $n \neq -1$

Thm. Change of variables:

If u=g(x) then $\int_a^b f(g(x))\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(u)\, du$

Thm. Integration of Odd & Even functions

If f is an even function on [-a,a], then

$$\int_{-a}^{a} f(x)\, dx = 2\int_{0}^{a} f(x)\, dx$$

If f is an odd function on [-a,a], then

$$\int_{-a}^{a} f(x)\, dx = 0$$

ex. $\int x\sqrt{(x+2)}\,dx$