4.5 Integration by Simple Substitution

Thm. Antiderivative of a Composite Function

$$\int f(g(x))g'(x)dx = F(g(x)) + C$$

or, if v=g(x), then dv=g '(x) dx and $\int f(v)dv = F(v) + C$

ex. $\frac{1}{9}\int 9x^2 (3x^3 + 2)^2\, dx \quad = \frac{1}{9}\int 9x^2 (3x^3+2)^2 dx$

$u = 3x^3 + 2$

$du = 9x^2\, dx$

$\frac{3(3x^3+2)^{\cancel{3}\,2}}{27} \cdot 9x^2 \; \cancel{+C}$

$= \frac{1}{9}\int u^2 \, du$

$= \frac{1}{9} \cdot \frac{u^3}{3} + C$

$= \frac{(3x^3+2)^3}{27} + C$

ex. $\int 5\cos(5x)\,dx \quad = \int \cos u \, du$

$u = 5x$

$du = 5\, dx$

$= \sin u + C$

$= \sin(5x) + C$

If the substitution $x = \cos\theta$ is made, then

$$\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$$

is equivalent to which one(s) of the following:

$$-\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} d\theta \qquad \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} d\theta \qquad \frac{\pi}{6}$$

If the substitution $u = 1 + \sqrt{x}$ is made, then $\int_0^1 \frac{\sqrt{x}}{1+\sqrt{x}}dx$

is equivalent to which one of the following:

$2\int_1^2 \frac{u-1}{u}du$ $\qquad$ $2\int_1^2 \frac{(u-1)^2}{u}du$ $\qquad$ $2\int_0^1 \left(1-\frac{1}{u}\right)du$

$\int_1^2 \left(2u-4+\frac{2}{u}\right)du$ $\qquad$ $2\int_0^2 \frac{(u-1)^2}{u}du$

If the substitution $x = 2\sin y$ is made, then $\int_0^2 x^3\sqrt{4 - x^2}\,dx$ can be re-written as...

ex. $\int x\sqrt{(x^2+1)}\,dx$ $= \frac{1}{2}\int 2x\sqrt{x^2+1}\,dx$

$u = x^2+1$

$du = 2x\,dx$

$= \frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\int u^{\frac{1}{2}}\,du$

$= \frac{1}{2}\cdot\frac{2}{3}\frac{u^{\frac{3}{2}}}{\cancel{\frac{3}{2}}} + C$

$= \frac{1}{3}(x^2+1)^{3/2} + C$

ex. $\frac{1}{2}\int \frac{-x\cdot 2}{(x^2-4)^3}\,dx$

$u = x^2-4$

$du = 2x\,dx$

$= -\frac{1}{2}\int \frac{2x\,dx}{(x^2-4)^3}$

$= -\frac{1}{2}\int \frac{du}{u^3} = -\frac{1}{2}\int u^{-3}\,du$

$= -\frac{1}{2}\cdot\frac{u^{-2}}{-2} + C = \frac{1}{4u^2} + C = \frac{1}{4(x^2-4)^2} + C$

by substitution, by change limits, by calculator

ex. $\frac{1}{4}\int_0^2 \frac{4x}{\sqrt{1+2x^2}}dx = \frac{1}{4}\int_{x=0}^{x=2} \frac{du}{\sqrt{u}} = \frac{1}{4}\int_{x=0}^{x=2} u^{-\frac{1}{2}}du$

$u = 1 + 2x^2$

$du = 4x\,dx$

$= \frac{1}{4}\cdot 2u^{\frac{1}{2}}\Big|_{x=0}^{x=2}$

$= \frac{1}{2}\sqrt{1+2x^2}\Big|_0^2$

$= \frac{1}{2}\sqrt{1+2\cdot 2^2} - \frac{1}{2}\sqrt{1+2\cdot 0^2}$

$= \frac{3}{2} - \frac{1}{2} = 1$

$= \frac{1}{4}\int_1^9 \frac{du}{\sqrt{u}}$

$= \frac{1}{4}\int_1^9 u^{-\frac{1}{2}}du$

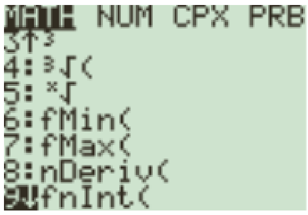
$= \frac{1}{4}\cdot 2u^{\frac{1}{2}}\Big|_1^9$

$= \frac{1}{2}\sqrt{u}\Big|_1^9 = \frac{1}{2}\sqrt{9} - \frac{1}{2}\sqrt{1} =$

$\frac{3}{2} - \frac{1}{2} = 1$

Definite Integrals on the calculator

Press the MATH button
Choose 9:fnInt(
and enter the following:



fnInt(function,variable, lower limit, upper limit)

Thm. The General Power Rule for Integration

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C \qquad \text{for } n \neq -1$$

Thm. Change of variables:

If u=g(x) then $\int_a^b f(g(x))\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(u)\, du$

Thm. Integration of Odd & Even functions

If f is an even function on [-a,a], then

$$\int_{-a}^{a} f(x)\, dx = 2\int_0^a f(x)\, dx$$

If f is an odd function on [-a,a], then

$$\int_{-a}^{a} f(x)\, dx = 0$$

ex. $\int x\sqrt{(x+2)}\,dx$

$u = x+2$

$du = 1\,dx$

$u-2 = x$

$= \int (u-2)\sqrt{u}\,du$

$= \int \left(u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right) du$

$= \frac{2}{5}u^{\frac{5}{2}} - 2\cdot\frac{2}{3}u^{\frac{3}{2}} + C$

$= \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$