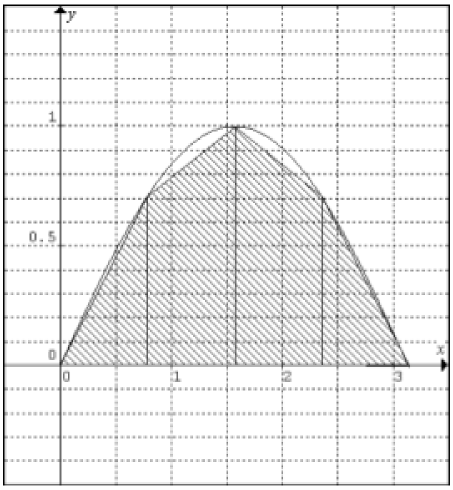
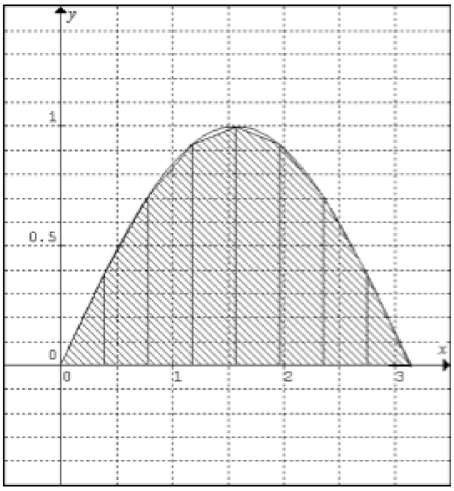
4.6 Numerical Integration

First Method: The Trapezoidal Rule

ex. Approximate $\int_0^{\pi} \sin x \, dx$ using n=4 and again with n=8.



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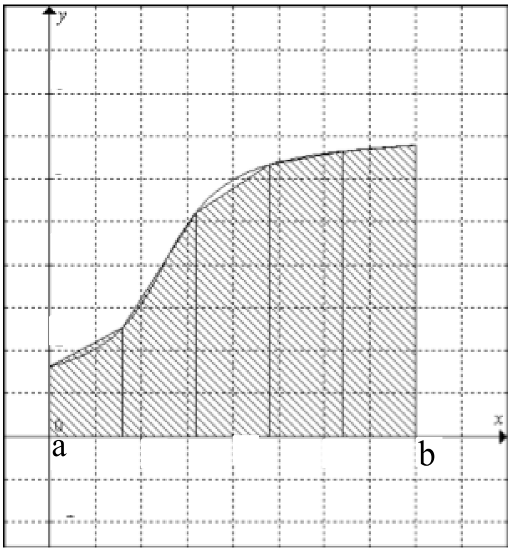
π(1+√(2))/4
        1.896118898
π/16*(2*.3827+2*
.7071+2*.9239+2*
1+2*.9239+2*.707
1+2*.3827)
        1.974255363
```



Recall that the area of a trapezoid is $A=\frac{1}{2}h(b_1+b_2)$

Height of each trapezoid: $h=\frac{b-a}{n}$

Lengths of bases of ith trapezoid: $f(x_i)$ and $f(x_{i-1})$ whe
$x_0 = a$ and $x_n = b$.

So, $\int_a^b f(x)dx$

$$=\frac{1}{2}\cdot\frac{b-a}{n}\Big[[f(x_0)+f(x_1)]+[f(x_1)+f(x_2)]+[f(x_2)+f(x_3)]+[f(x_3)+f(x_4)]+[f(x_4)+f(x_5)]\Big]$$

The Trapezoidal Rule

$$\int_a^b f(x)dx = \frac{b-a}{2n}[f(x_0)+2f(x_1)+ 2f(x_2)+2f(x_3)+...+2f(x_{n-1})+f(x_n)]$$

where $a=x_0$ and $b=x_n$

ex. Estimate $\int_1^2 \frac{1}{x}\,dx$

using the Trapezoidal Rule with n=4.

Error:

1. The error in approximating $\int_a^b f(x)dx$ using the Trapezoidal Rule is: $E \leq \frac{(b-a)^3}{12n^2}[\max |f''(x)|]$ for x in [a,b].

ex. For $\int_1^2 \frac{1}{x}\,dx$, find the max error estimate for the Trapezoidal Rule using n=4.

Approximate $\int_1^3 \sqrt{1+x^3}\,dx$ with 4 equal intervals

a. using a left approximation

b. using a right approximation

c. using a midpoint approximation

d. using a trapezoid approximation

Optional method (not on the AP exam)

Thm. Simpson's Rule (n must be even)

$$\int_a^b f(x)dx$$

$$=\frac{b-a}{3n}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+\ldots+4f(x_{n-1})+f(x_n)]$$

ex. Use Simpson's Rule to approximate $\int_0^1 x^3\,dx$ with n=4.

2. The error in approximating $\int_a^b f(x)dx$

using Simpson's Rule is: $E \leq \frac{(b-a)^5}{180n^4}[\max |f^{(4)}(x)|]$

for x in [a,b].