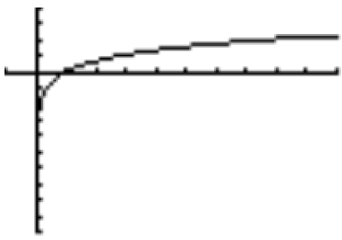
5.1 The Natural Log Function and Differentiation

Defn. $\int_1^x \frac{1}{t}\,dt = \ln x$ given that x>0.

Thm. Properties of ln

1. Domain: (0,∞) Range: (-∞,∞)
2. Increasing function
3. Continuous function
4. 1-1 function (passes the horizontal line test)
5. Concave down

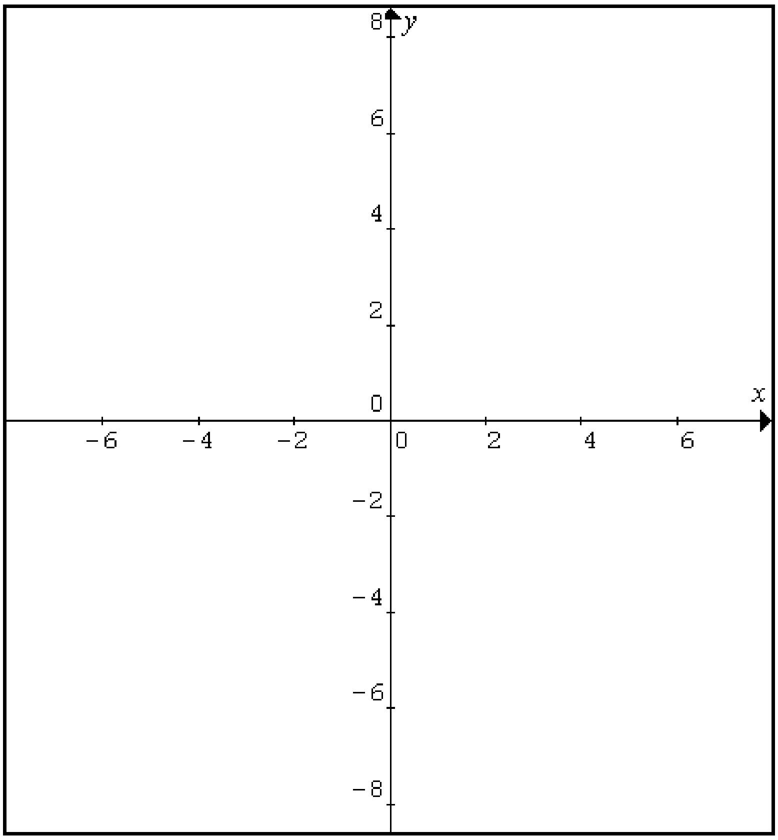


6. $\ln 1 = 0$
7. $\ln (ab) = \ln a + \ln b$
8. $\ln \left(\frac{a}{b}\right) = \ln a - \ln b$
9. $\ln a^b = b \ln a$
10. $\ln x = y \text{ iff } e^y = x$

ex. Write an equivalent expression. $\ln \sqrt{3x+9}$

ex. Write an equivalent expression. $2\ln x - 3\ln y + \frac{1}{2}\ln z$

ex. Graph y=lnx +1
Give the domain.

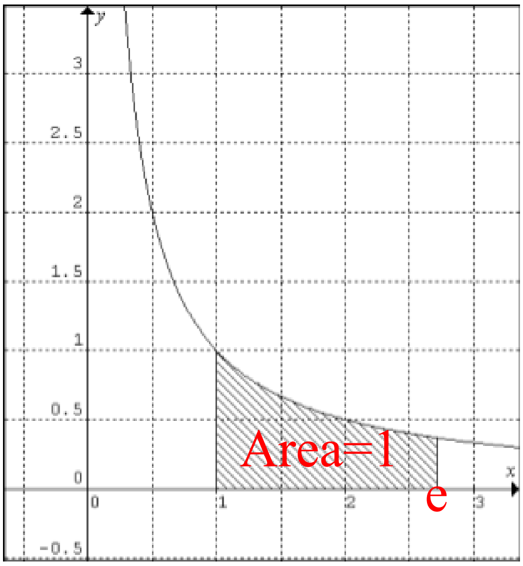


ex. Graph y=ln(x-1)
Give the domain.

ex. Calculator active problem:
If $f'(x)=\sin(\ln x)$, how many relative extrema does f have in the interval (0,2]?

Defn. The letter e denotes the positive real number such that

$$\ln e = \int_1^e \frac{1}{t}\,dt = 1$$



e ≈ 2.71828182846

A bit of trivia:
e is called the Euler number (pronounced "OY-lur") for the Swiss mathematician Leonhard Euler.

Thm. Let u be a differentiable function of x, then

1. $\frac{d}{dx}[\ln x] = \frac{1}{x}$

2. $\frac{d}{dx}[\ln u] = \frac{1}{u} \bullet \frac{du}{dx} = \frac{u'}{u}$

ex. If y=ln(cos(2x)), then y'=

ex. The total energy expenditure per day (in excess of growth) for a species of fawn is given by

$$E(x) = 0.774 + 0.727\ln(x)$$

where x is the fawn's mass in grams and E(x) is the energy expenditure in kJ/day. Find and interpret E'(10,000), including units.

ex. Write an equation of the line tangent to the graph of
$y = 2x^2 + \ln(3x - 8)$ at the point where x=3.

ex. If tan x= ln(xy), find dy/dx.

ex. Find when a particle is at rest if its position on the x axis at time t is $x(t) = t \ln t$

ex. Find x coordinates of any points of inflection of

$$f(x) = \ln\left(x^2 + 2\right)$$

ex. Though not tested on the AP exam, the process called logarithmic differentiation can be helpful:

$$y=\frac{(x-2)^2}{\sqrt{x^2+1}}$$

I'll need this later, after I extend the page, so I put it here again.

$$y=\frac{(x-2)^2}{\sqrt{x^2+1}}$$