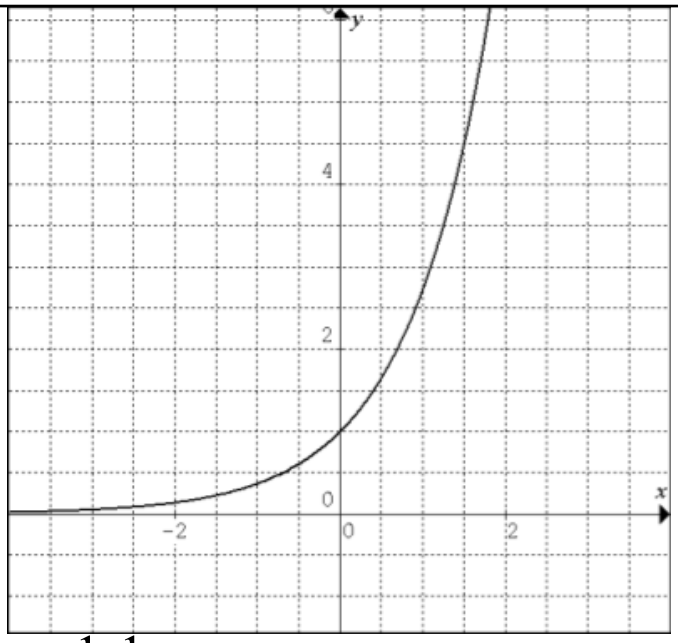
5.4 Exponential Functions: Differentiation and Integration

Defn. The inverse of the natural log function f(x)=lnx is called the natural exponential function $f^{-1}(x)=e^x$
That is, y= e^x iff lny = x.

ex. Solve for x: $6=e^{2x}$

ex. graph of y= e^x



Fact: $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$

Note:
1. Domain: $(-\infty,\infty)$
2. Range: $(0,\infty)$
3. monotonic increasing, hence 1-1
4. Continuous and concave up
5. $\lim_{x\to-\infty} e^x = 0$
6. $\lim_{x\to\infty} e^x = \infty$

Thm.
1. $e^a e^b = e^{a+b}$
2. $\frac{e^a}{e^b} = e^{a-b}$

ex. Find f '(x) for f(x)= e^x by using logarithmic differentiation.

Thm.

1. $\frac{d}{dx}e^x = e^x$
2. $\frac{d}{dx}e^u = e^u \cdot u' = e^u \frac{du}{dx}$

ex. Find f '(x) for $f(x) = e^{\sin x}$

ex. Find x coordinates of all extrema of

$$f(x) = x^2 e^{-2x}$$

ex. $\frac{d}{dx}\left[\ln(x+e^{x^2})\right]$

ex. At what point does this function have a horizontal tangent line?

$$f(x) = \frac{e^x}{x}$$

Thm.

1. $\int e^x dx = e^x + C$

2. $\int e^u du = e^u + C$

ex. $\int e^{3x+1} dx$

ex. Use a calculator to find the average height above the x axis of $y = e^{-x^2}$ on the interval [-3,3].

ex. $\int \frac{e^{3x}}{1+e^{3x}} dx$

ex. $\lim_{x \to 0} \dfrac{e^{3+x} - e^3}{x}$