5.4 Exponential Functions: Differentiation and Integration

Defn. The inverse of the natural log function f(x)=lnx is called the natural exponential function $f^{-1}(x)=e^x$

That is, y= e^x iff lny = x.

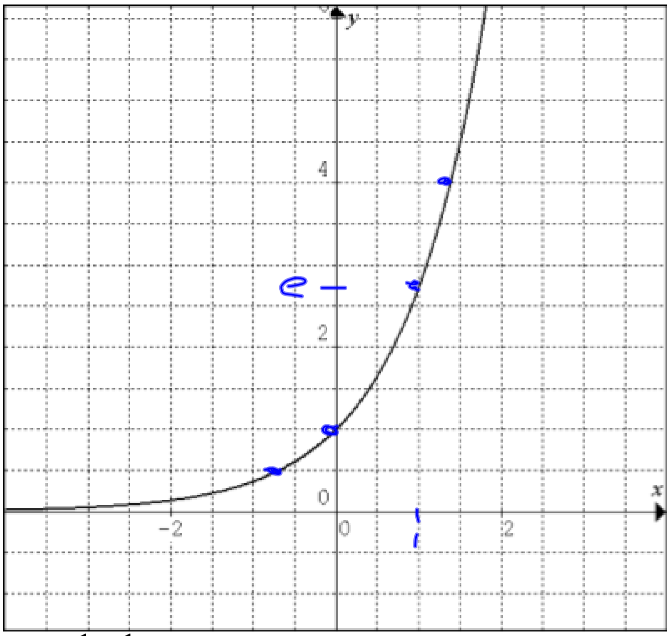
ex. Solve for x: $6=e^{2x}$

$\ln 6 = \cancel{\ln} e^{2x}$

$\ln 6 = 2x$

$\frac{\ln 6}{2} = x$

ex. graph of y= e^x



Fact: $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$

Note:
1. Domain: $(-\infty,\infty)$
2. Range: $(0,\infty)$
3. monotonic increasing, hence 1-1
4. Continuous and concave up
5. $\lim_{x\to-\infty} e^x = 0$
6. $\lim_{x\to\infty} e^x = \infty$

Thm.
1. $e^a e^b = e^{a+b}$
2. $\frac{e^a}{e^b} = e^{a-b}$

ex. Find f '(x) for f(x)= e^X by using logarithmic differentiation.

$$y = e^x$$

$$\ln y = \cancel{\ln} e^x$$

$$\ln y = x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = y$$

$$\frac{dy}{dx} = e^x$$

Thm.

1. $\frac{d}{dx}e^x = e^x$

2. $\frac{d}{dx}e^u = e^u \cdot u' = e^u \frac{du}{dx}$

ex. Find f '(x) for $f(x) = e^{\sin x}$

$f'(x) = (e^{\sin x})(\cos x)$

ex. Find x coordinates of all extrema of

$$f(x) = \underline{x^2}\,\underline{e^{-2x}}$$

$f'(x) = 2xe^{-2x} + x^2e^{-2x}\cdot(-2)$

$0 = \underline{2x}\,\underline{e^{-2x}}(1 - x)$

$2x = 0$ $\quad$ $1 - x = 0$

$x = 0$ $\quad$ $x = 1$

$f'(-1) < 0$ $\quad$ $f'(\frac{1}{2}) > 0$ $\quad$ $f'(2) < 0$

ex. $\frac{d}{dx}\left[\ln(x+e^{x^2})\right] = \frac{1}{x+e^{x^2}}\left(\frac{1+e^{x^2}\cdot 2x}{1}\right)$

$= \frac{1+2xe^{x^2}}{x+e^{x^2}}$

ex. At what point does this function have a horizontal tangent line?

$$f(x) = \frac{e^x}{x}$$

$$f'(x) = \frac{e^x \cdot x - e^x \cdot 1}{x^2}$$

$$0 = xe^x - e^x$$

$$0 = e^x\underbrace{(x - 1)}$$

$$x - 1 = 0$$

when $x = 1$

Thm.

1. $\int e^x dx = e^x + C$

2. $\frac{1}{3}\int e^u du = e^u + C$

$\frac{1}{\cancel{n}} si\cancel{n}x = ?$

$six = 6$

$\frac{ta\cancel{n}\,\cancel{x}}{si\cancel{n}\,\cancel{x}} = \frac{ta}{si}$

ex. $\int e^{3x+1} dx = \frac{1}{3}\int \underline{3}e^{3x+1}\,\underline{dx}$

$u = 3x+1$
$du = 3\,dx$

$= \frac{1}{3}\int e^u\,du = \frac{1}{3}e^u + C$

$= \frac{1}{3}e^{3x+1} + C$

ex. Use a calculator to find the average height above the x axis of $y = e^{-x^2}$ on the interval [-3,3].

$\frac{\text{integral}}{\text{interval}}$

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fnInt(e^(-X²),X,
-3,3)/(3--3)
        .2954024494
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ex. $\frac{1}{3}\int \frac{3e^{3x}}{1+e^{3x}}dx = \frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln|u| + C$

$= \frac{1}{3}\ln|1+e^{3x}| + C$

$u = 1 + e^{3x}$

$du = e^{3x} \cdot 3 = 3e^{3x}\,dx$

ex. $\lim_{x \to 0} \frac{e^{3+x} - e^{3+0}}{x - 0}$

$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

$f(x) = e^{3+x}$ $f'(x) = e^{3+x}$

$f'(0) = ?$ $f'(0) = e^{3+0} = \boxed{e^3}$