

5.5 Bases other than e and applications

Recall your exponent rules:

For a positive real number a , $a \neq 1$, and any real x :

$$1. a^0 = 1 \qquad 2. a^x a^y = a^{x+y}$$

$$3. a^x / a^y = a^{x-y} \qquad 4. (a^x)^y = a^{xy}$$

Definition of \log_a (Log base a)

$$\log_a x = \frac{\ln x}{\ln a}$$

Properties:

$$1. \log_a 1 = 0 \qquad 2. \log_a xy = \log_a x + \log_a y$$

$$3. \log_a x / y = \log_a x - \log_a y \qquad 4. \log_a x^y = y \log_a x$$

Properties of log and its inverse

1. $\log_a x = y$ iff $a^y = x$

2. $\log_a a^x = x$

3. $a^{\log_a x} = x$

ex. Solve $3^x = \frac{1}{81}$

ex. Solve $\log_2 x = -4$

ex. Solve $\log_2 64 = 3x + 5$

Derivatives

$$1. \frac{d}{dx} [a^x] = \ln a \cdot a^x$$

here's how: \longrightarrow

$$\begin{aligned} y &= a^x \\ \ln y &= x \ln a \\ \frac{y'}{y} &= 1 \cdot \ln a \\ y' &= y \cdot \ln a \\ y' &= a^x \cdot \ln a \end{aligned}$$

$$2. \frac{d}{dx} [a^u] = \ln a \cdot a^u \cdot u'$$

$$3. \frac{d}{dx} [\log_a x] = \frac{d}{dx} \left[\frac{\ln x}{\ln a} \right] = \frac{1}{x \ln a}$$

$$4. \frac{d}{dx} [\log_a u] = \frac{u'}{u \ln a}$$

Integral formula

$$\int a^x dx = \frac{1}{\ln a} \cdot a^x + C$$

ex. If $y = 2^{3x^2}$, then $y' =$

ex. If $f(x) = \log_{10} \frac{x^2}{x-1}$, then $f'(x) =$

$$\text{ex. } \int x 7^{3x^2} dx =$$