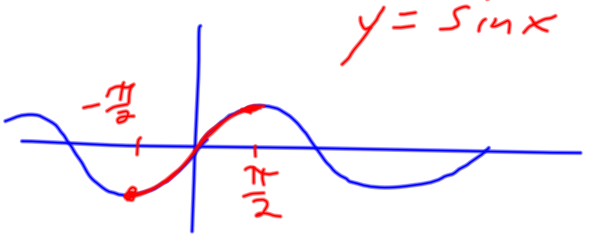
5.6 Inverse Trig Functions and Differentiation

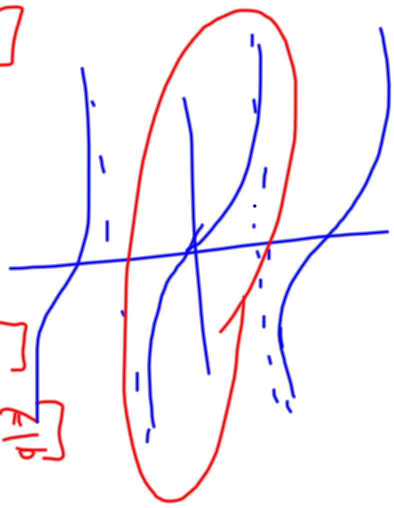
Note: $\arcsin x = \sin^{-1} x \neq (\sin x)^{-1} = \csc x$

	Domain of y	Range of y
y=arcsinx iff siny=x	[-1,1]	[-π/2,π/2]
y=arccosx iff cosy=x	[-1,1]	[0,π]
y=arctanx iff tany=x	(-∞,∞)	(-π/2,π/2)
y=arccotx iff coty=x	(-∞,∞)	(0,π)
y=arcsecx iff secy=x	(-∞,-1]∪[1,∞)	[0,π/2)∪(π/2,π]
y=arccscx iff cscy=x	(-∞,-1]∪[1,∞)	[-π/2,0)∪(0,π/2]



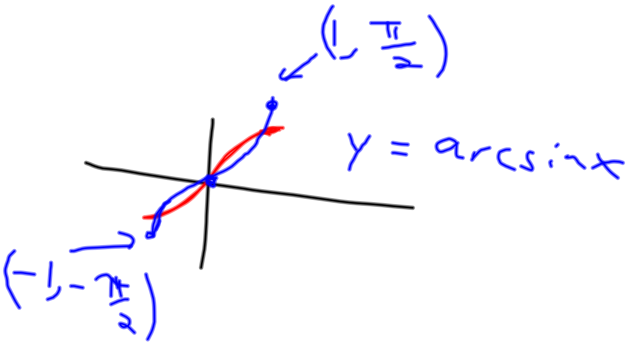
$y = \sin x$ $\quad D: [-\frac{\pi}{2}, \frac{\pi}{2}]$

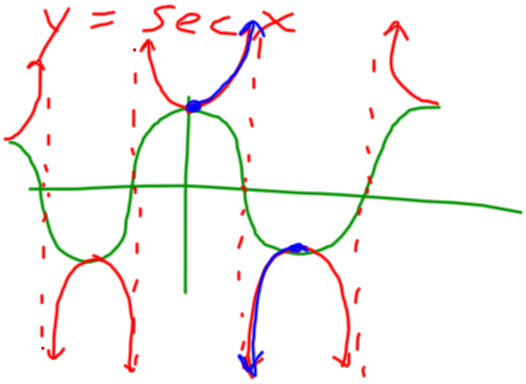
$R: [-1, 1]$



$y = \arcsin x$ $\quad D: [-1, 1]$

$R: [-\frac{\pi}{2}, \frac{\pi}{2}]$

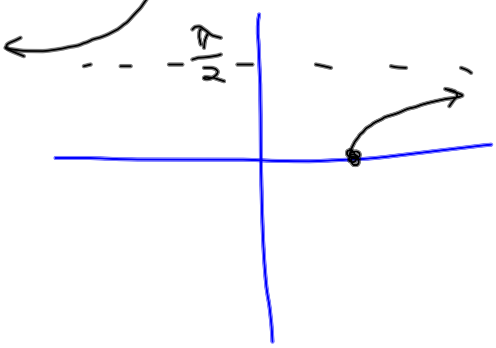




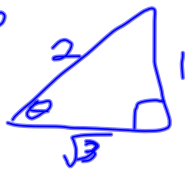
D: $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ R

R: $(-\infty, -1] \cup [1, \infty)$ D

$y = \operatorname{arcsec} x$

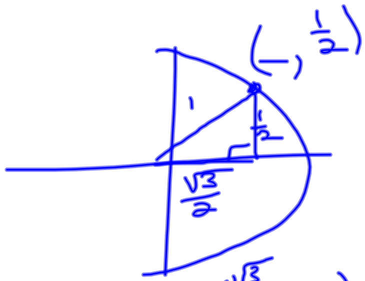


ex. $\arcsin \frac{1}{2}$ opp / hyp

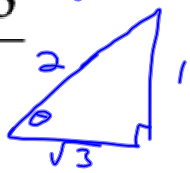


30°

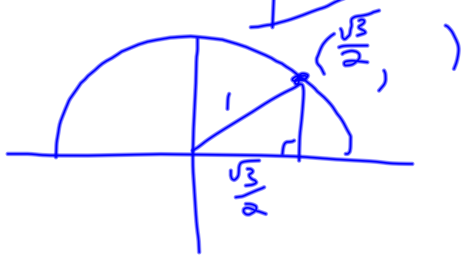
$\frac{\pi}{6}$



ex. $\arccos \frac{\sqrt{3}}{2}$ adj

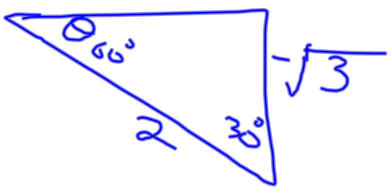


$\frac{\pi}{6}$

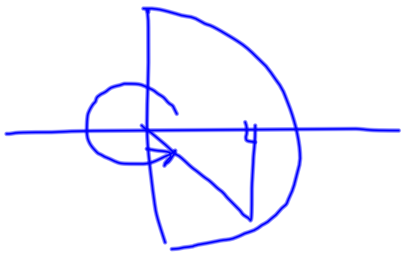


ex. $\arctan\left(-\sqrt{3}\right)$ opp

$\frac{-\sqrt{3}}{1}$ adj



$-\frac{\pi}{3}$



ex. $\cancel{\arctan}(2x-3) \overset{\tan}{=} \frac{\pi}{4}$

tan

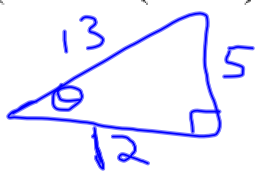
$2x - 3 = \tan \frac{\pi}{4}$

$2x - 3 = 1$

$2x = 4$

$x = 2$

ex. $\cos(\arcsin(5/13)) = \frac{12}{13}$

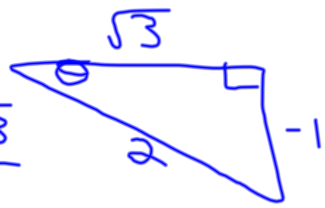


ex. $\cot(\arcsin(-1/2))$

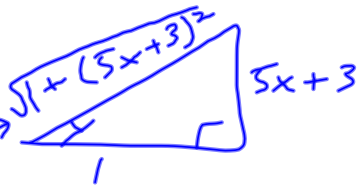
$\frac{adj}{opp}$

$= \frac{\sqrt{3}}{-1}$

$= -\sqrt{3}$



ex. Find y' if $y = \arctan(5x + 3)$



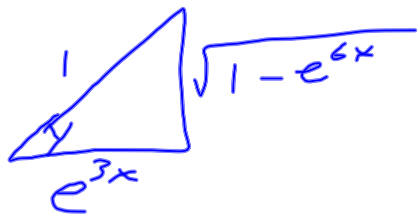
$$\tan y = \frac{5x+3}{1}$$

$$\sec^2 y \cdot \frac{dy}{dx} = 5$$

$$\frac{dy}{dx} = \frac{5}{\sec^2 y} = 5 \cdot \cos^2 y = 5(\cos y)^2$$

$$= 5\left(\frac{1}{\sqrt{1+(5x+3)^2}}\right)^2 = 5\left(\frac{1}{1+(5x+3)^2}\right)$$

ex. Find y' if $y = \arccos(e^{3x})$



$$\cos y = \frac{e^{3x}}{1}$$

sohcahtoa

$$-\sin y \frac{dy}{dx} = e^{3x} \cdot 3$$

$$\frac{dy}{dx} = \frac{3e^{3x}}{-\sin y} = -\frac{3e^{3x}}{\frac{\sqrt{1-e^{6x}}}{1}} = -\frac{3e^{3x}}{\sqrt{1-e^{6x}}}$$

Derivatives of inverse trig functions

$$\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}\operatorname{arccsc} u = \frac{-u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx}\arccos u = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}\operatorname{arcsec} u = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx}\arctan u = \frac{u'}{1+u^2}$$

$$\frac{d}{dx}\operatorname{arccot} u = \frac{-u'}{1+u^2}$$