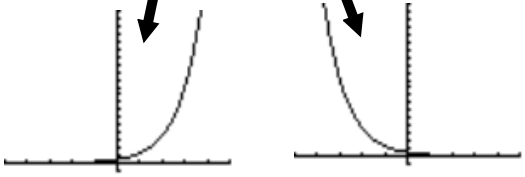
5.8 Hyperbolic Functions

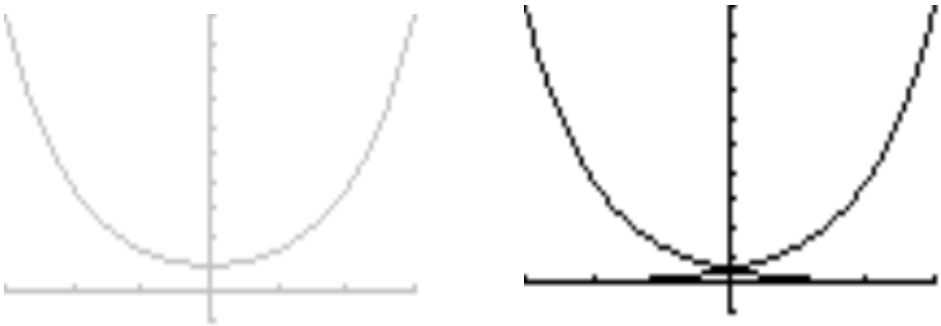
Defn. $\sinh x = \frac{e^x - e^{-x}}{2}$ $\quad \operatorname{csch} x = \frac{1}{\sinh x}$, $x \neq 0$

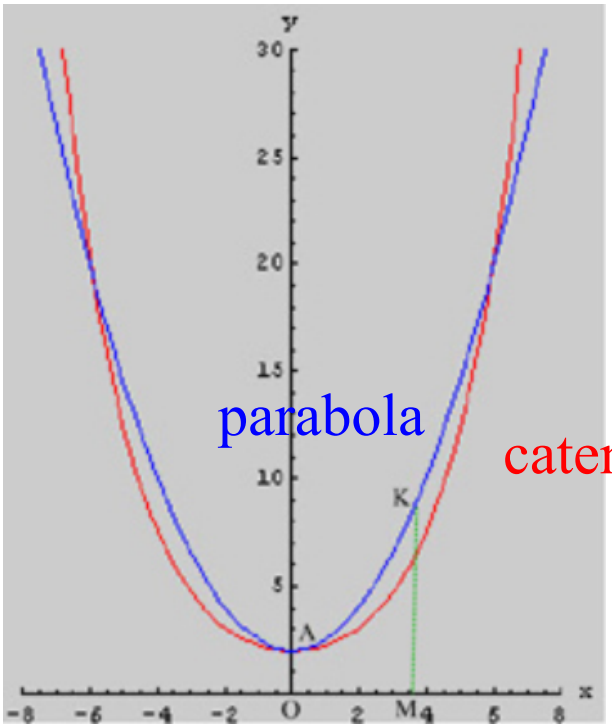
$\cosh x = \frac{e^x + e^{-x}}{2}$ $\quad \operatorname{sech} x = \frac{1}{\cosh x}$

$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ $\quad \coth x = \frac{1}{\tanh x}$, $x \neq 0$

$$y = \cosh(x) = \frac{e^x + e^{-x}}{2} = \frac{e^x}{2} + \frac{e^{-x}}{2}$$







parabola

catenary

ex. $\frac{d}{dx}\sinh x$

Derivative formulas:

$\frac{d}{dx}\ \text{sinhu} = \text{coshu}\ u'$

$\frac{d}{dx}\ \text{coshu} = \text{sinhu}\ u'$

$\frac{d}{dx}\ \text{tanhu} = \text{sech}^2 u\ u'$

$\frac{d}{dx}\ \text{cothu} = -\text{csch}^2 u\ u'$

$\frac{d}{dx}\ \text{sechu} = -\text{sechu tanhu}\ u'$

$\frac{d}{dx}\ \text{cschu} = -\text{cschu cothu}\ u'$

ex $\frac{d}{dx}\sinh(x^2-3)$

ex $\frac{d}{dx}\ln(\cosh x^2)$

ex. Find the extrema of $f(x)=(x-1)\cosh x-\sinh x$

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ex. $\int \frac{dx}{x\sqrt{4-9x^2}}$

ex. $\frac{d}{dx}$:anh (x/2)

ex. $\int \frac{dx}{5-4x^2}$