6.1 Slope Fields and Euler's Method

Remember: Solutions of differential equations are functions like $y=f(x)+C$

The order of a DE is the highest order derivative of the equation.

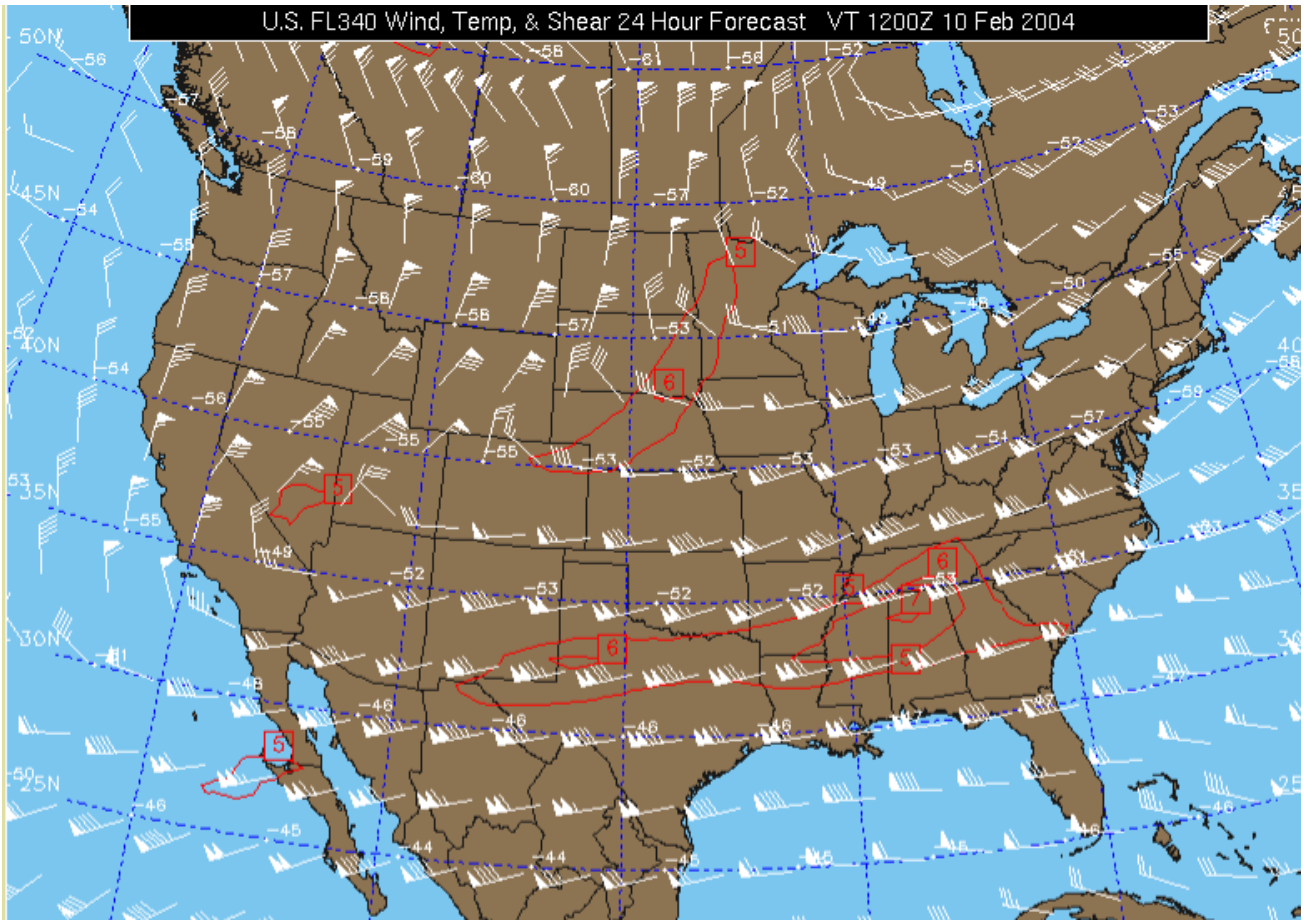
Check to see if these functions are solutions for the second order differential equation y" - y = 0.

a. $y = \sin x$

b $y = 5x^7$

c $y = 4e^{-x}$

d. $y = Ce^{-x}$


U.S. FL340 Wind, Temp, & Shear 24 Hour Forecast VT 1200Z 10 Feb 2004

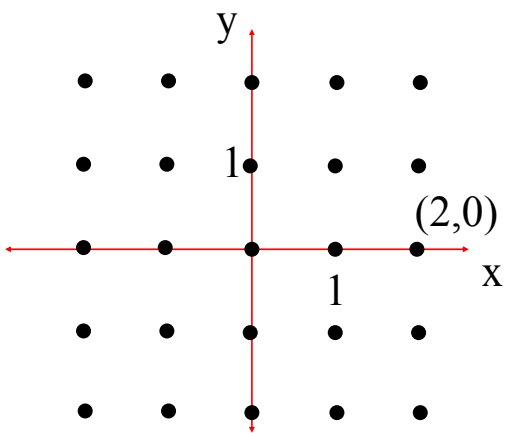
http://www.slu.edu/classes/maymk/Applets/IntegralCurves.html

GNAW on Differential Equations
Graphically: slope fields

Differential Equation (DE)
ex. draw the slope field for

$$\frac{dy}{dx} = -\frac{x}{y}$$

at the indicated points



Along the x axis, use vertical dashes except at the origin, since it is an indeterminate form, having different values for each different particular solution.

Another option-- you're free to draw nearly-vertical segments on each side of the x axis.

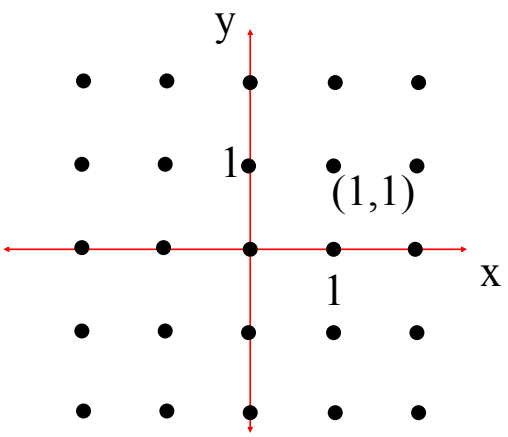
Initial Value Problem (IVP)
ex. draw the solution curve for this DE if x=2 when y=0

GNAW on Differential Equations
Graphically: slope fields

Differential Equation (DE)
ex. draw the slope field for

$$\frac{dy}{dx} = x + y$$

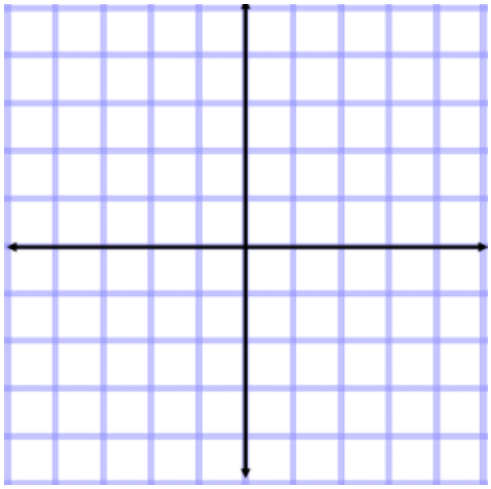
at the indicated points



Initial Value Problem (IVP)
ex. draw the solution curve
for this DE if x=1 when y=1

Analytically: Find the antiderivative to get a general solution and then graph 3 particular solutions of

$$\frac{dy}{dx} = 2x$$



GNAW on Differential Equations

BC Numerically: Euler's method

ex. Estimate f(1.2) given f(1)=4 and a step size of 0.1, if $f'(x) = -\sqrt{y}$

x	y	y '
1	4	-2
1.1	3.8	*
1.2	**	

Make a table and complete it like this:

A. $y' = -\sqrt{4} = -2$

B. $\frac{y-4}{1.1-1} = -2 \rightarrow y = 3.8$

C. $y' = -\sqrt{3.8} = -1.94936*$

D. $\frac{y-3.8}{1.2-1.1} = -1.94936 \rightarrow y = 3.605**$

GNAW on Differential Equations

BC Numerically: Euler's method

Again with more detail:

Euler's Method for finding a numerical estimate for a solution to an IVP (can be used to graph an estimated (piecewise linear) solution)

1. Start: initial condition (x,y).
2. Using DE, find slope=dy/dx.
3. Change x by Δx. Change y by $\Delta y = (dy/dx)\Delta x$. Get new point $(x+\Delta x, y+\Delta y)$ on the tangent line, the linearization of the solution curve.
4. Using new point & repeat step 2.
5. Repeating constructs an approximate solution to the right of initial point.

Negative values for Δx constructs an approximate solution to left of initial point.

BC Numerically: Euler's method

Initial Value Problem (IVP)

ex. Use Euler's method to estimate the value of f(1.2) if the function y=f(x) satisfies the differential equation $dy/dt = -y^{1/2}$ and f(1)=4. Use $\Delta t=0.1$ for your step size.

t	y	y'
1	4	
1.1		
1.2		

ex. Let f be a function that satisfies the IVP in $dy/dx=x+y$ and $f(2)=0$. Use Euler's method and increments of $\Delta x=0.2$ to approximate f(3).

t	y	y'
2	0	
2.2		
2.4		
2.6		
2.8		

Attachments

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