6.1 Slope Fields and Euler's Method

Remember: Solutions of differential equations are functions like y=f(x)+C

The order of a DE is the highest order derivative of the equation.

Check to see if these functions are solutions for the second order differential equation y" - y = 0.

a. $y = \sin x$ $y'' = -\sin x$

$y' = \cos x$ $-\sin x - \sin x \neq 0$

no

b $y = 5x^7$ $y'' = 210x^5$ $210x^5 - 5x^7 \neq 0$

$y' = 35x^6$

yes

c $y = 4e^{-x}$ $4e^{-x} - 4e^{-x} = 0$

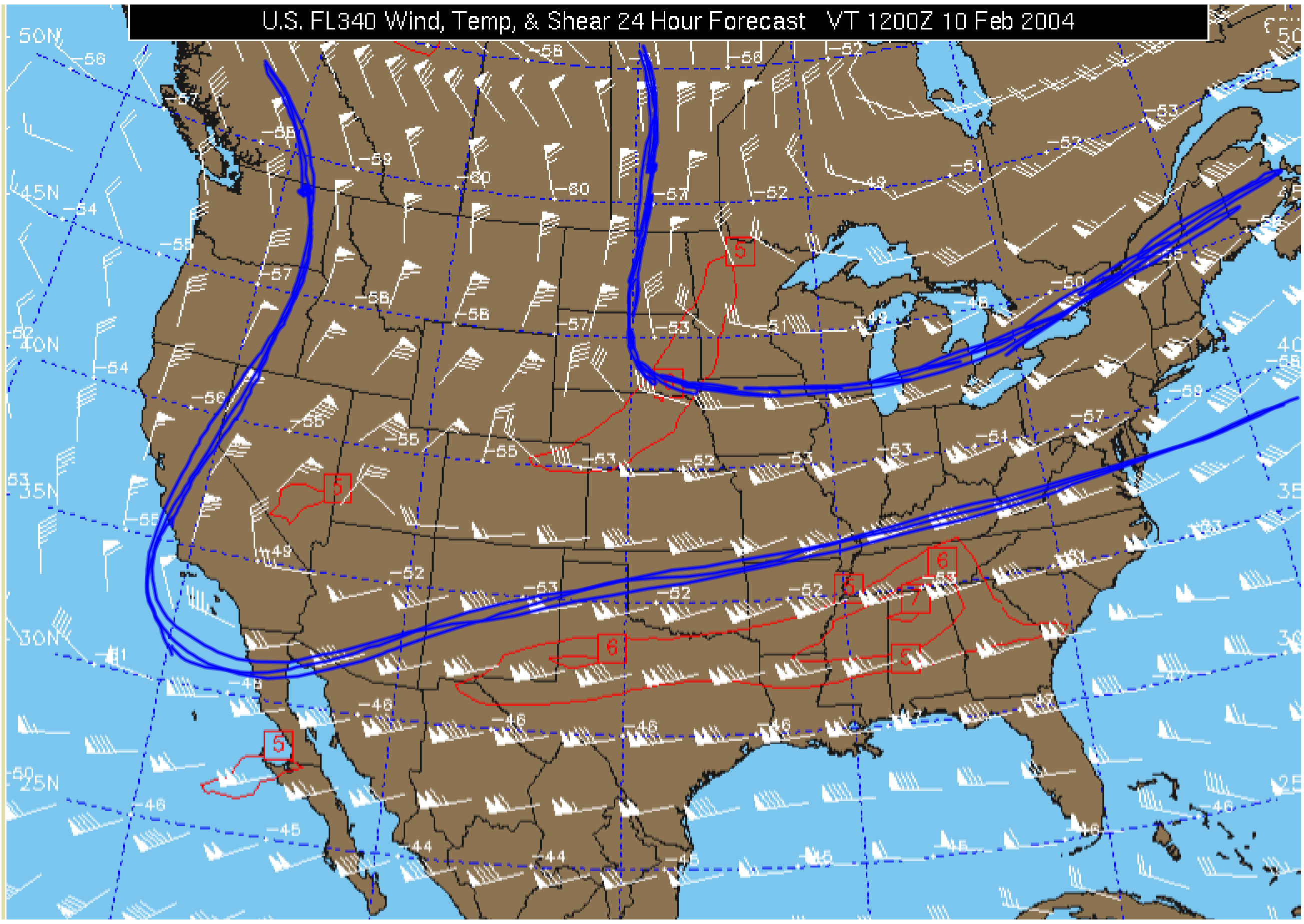
$y' = -4e^{-x}$ no

$y'' = 4e^{-x}$

d. $y = Ce^{-x}$

$y' = -Ce^{-x}$ $Ce^{-x} - Ce^{-x} = 0$

$y'' = Ce^{-x}$ yes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http://www.slu.edu/classes/maymk/Applets/IntegralCurves.html

GNAW on Differential Equations
Graphically: slope fields

Differential Equation (DE)
ex. draw the slope field for

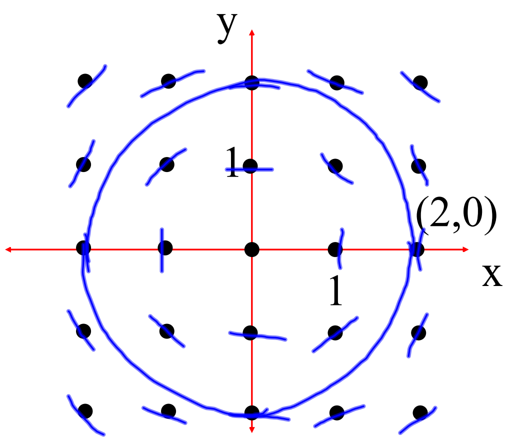
$$\frac{dy}{dx} = -\frac{x}{y}$$

at the indicated points

$$\int y\,dy = -\int x\,dx$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + C$$

$$\frac{y^2}{2} + \frac{x^2}{2} = C$$



Along the x axis, use vertical dashes except at the origin, since it is an indeterminate form, having different values for each different particular solution.

Another option-- you're free to draw nearly-vertical segments on each side of the x axis.

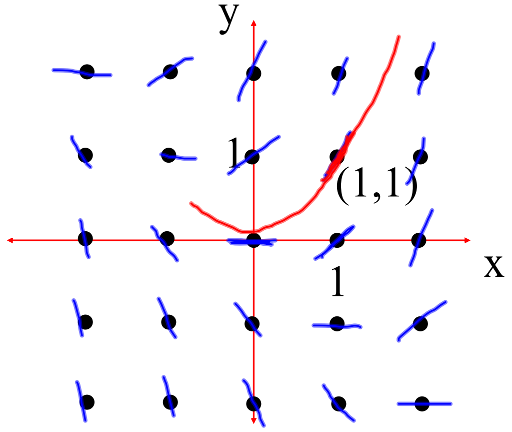
Initial Value Problem (IVP)
ex. draw the solution curve for this DE if x=2 when y=0

GNAW on Differential Equations
Graphically: slope fields

Differential Equation (DE)
ex. draw the slope field for

$$\frac{dy}{dx} = x + y$$

at the indicated points



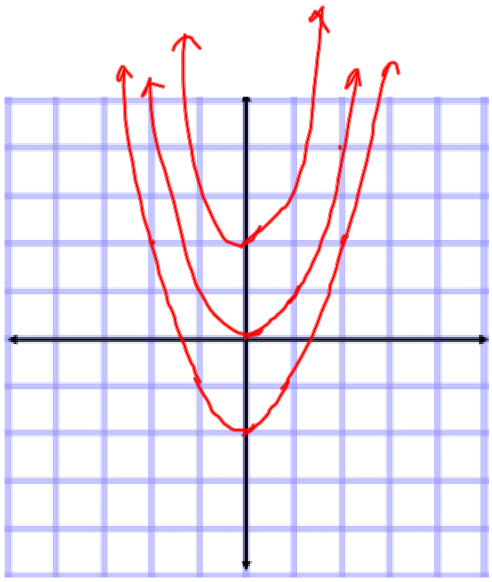
Initial Value Problem (IVP)
ex. draw the solution curve
for this DE if x=1 when y=1

Analytically: Find the antiderivative to get a general solution and then graph 3 particular solutions of

$$\frac{dy}{dx} = 2x$$

$$\int dy = \int 2x\ dx$$

$$y = x^2 + C$$



GNAW on Differential Equations

BC Numerically: Euler's method

ex. Estimate f(1.2) given f(1)=4 and a step size of 0.1, if $f'(x) = -\sqrt{y}$

x	y	y '
1	4	-2
1.1	3.8	*
1.2	**	

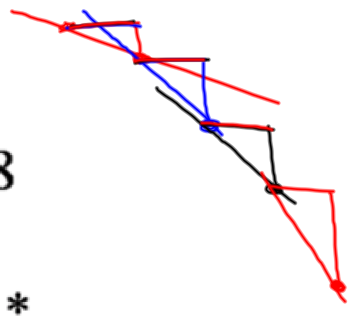
Make a table and complete it like this:

A. $y' = -\sqrt{4} = -2$

B. $\dfrac{y-4}{1.1-1} = -2 \rightarrow y = 3.8$

C. $y' = -\sqrt{3.8} = -1.94936*$

D. $\dfrac{y-3.8}{1.2-1.1} = -1.94936 \rightarrow y = 3.605**$



GNAW on Differential Equations

BC Numerically: Euler's method

Again with more detail:

Euler's Method for finding a numerical estimate for a solution to an IVP (can be used to graph an estimated (piecewise linear) solution)

1. Start: initial condition (x,y).
2. Using DE, find slope=dy/dx.
3. Change x by Δx. Change y by $\Delta y = (dy/dx)\Delta x$. Get new point $(x+\Delta x, y+\Delta y)$ on the tangent line, the linearization of the solution curve.
4. Using new point & repeat step 2.
5. Repeating constructs an approximate solution to the right of initial point.

Negative values for Δx constructs an approximate solution to left of initial point.

BC Numerically: Euler's method

Initial Value Problem (IVP)

ex. Use Euler's method to estimate the value of f(1.2) if the function y=f(x) satisfies the differential equation $dy/dt = -y^{1/2}$ and f(1)=4. Use Δt=0.1 for your step size.

t	y	y'
1	4	
1.1		
1.2		

ex. Let f be a function that satisfies the IVP in dy/dx=x+y and f(2)=0. Use Euler's method and increments of Δx=0.2 to approximate f(3).

x	y	y'
2	0	2
2.2	.4	2.6
2.4	.92	3.32
2.6	1.584	4.184
2.8	2.4208	5.2208
3		

$y' = 2 + 0 = 2$ $\quad \frac{y - 0}{2.2 - 2} = 2 \rightarrow y = .4$

$y' = 2.2 + .4 = 2.6$ $\quad \frac{y - .4}{2.4 - 2.2} = 2.6 \rightarrow y = .92$

$y' = 2.4 + .92 = 3.32$ $\quad \frac{y - .92}{2.6 - 2.4} = 3.32 \rightarrow y = 1.584$

$\frac{y - 1.584}{.2} = 4.184 \rightarrow y = 2.4208$

$\frac{y - 2.4208}{.2} = 5.2208$

$\rightarrow y = 3.46496$

3.464

3.465

Attachments

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