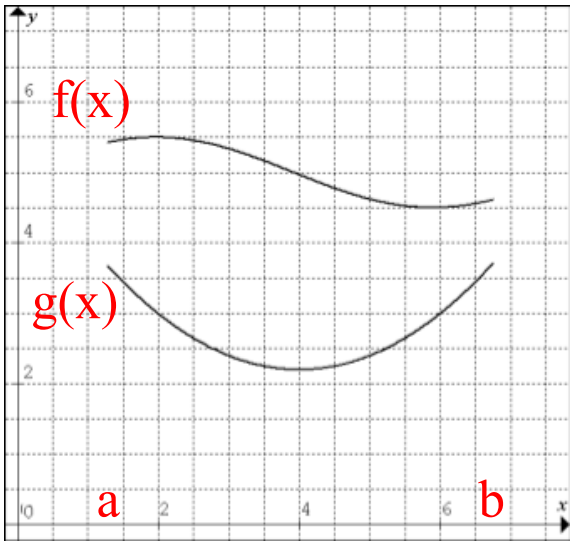
7.1 Area between two curves

Area under f

$$\int_a^b f(x)dx$$

Area under g

$$\int_a^b g(x)dx$$

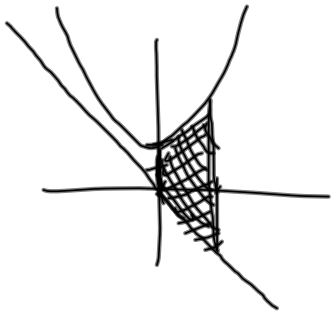


Area between f and g, as long as f and g are continuous on [a,b] and $f(x) \geq g(x)$.

$$\int_a^b [f(x)-g(x)]dx$$

Always remember: Top-Bottom and Right-Left

ex. Find the area between $y=x^2+2$, $y=-x$, $x=0$, and $x=1$.



$$\int_0^1 (x^2+2)-(-x)\,dx$$

$$= \int_0^1 x^2+x+2\,dx$$

$$= \left(\frac{x^3}{3}+\frac{x^2}{2}+2x\right)\Big|_0^1$$

$$= \frac{1}{3}+\frac{1}{2}+2 = 2\frac{5}{6}$$

ex. Use the calculator to find the area of the region bounded by $f(x) = \sin x$ and $g(x) = 4 - x^2$

Y Y

$f(x) = g(x)$

when $x = a = -2.193$
-2.194

and when
$x = b = 1.735$
1.736

$\int_a^b (4 - x^2) - \sin x \, dx$

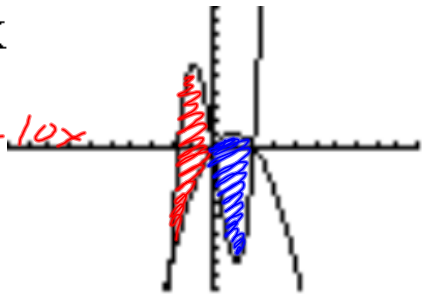
$\int_a^b g(x) - f(x) \, dx$

$= 10.874$
10.875

ex. Find the area between $y_1 = 3x^3 - x^2 - 10x$ and $y_2 = -x^2 + 2x$

Let $y_1 = 3x^3 - x^2 - 10x$

& $y_2 = -x^2 + 2x$



$Y_1 - Y_2$ $Y_2 - Y_1$

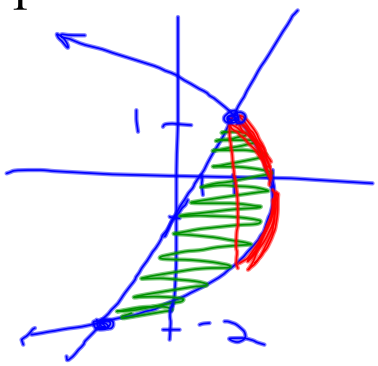
Y_1 & Y_2 intersect when $x = a = -2$, and $x = b = 0$, $x = c = 2$

$$\text{area} = \int_a^b (Y_1 - Y_2)\,dx + \int_b^c (Y_2 - Y_1)\,dx$$

$$= 12 + 12$$

$$= 24$$

ex. Find the area between $x = 3-y^2$ and $x = y+1$



$$\int_{-2}^{1} (3-y^2) - (y+1)\,dy$$

$$= \left(3y - \frac{y^3}{3} - \frac{y^2}{2} - y\right)\Big|_{-2}^{1}$$

$$= \left(3 - \frac{1}{3} - \frac{1}{2} - 1\right) - \left(-6 + \frac{8}{3} - 2 + 2\right)$$

$$1\frac{1}{6} + 10\frac{1}{3}$$

$$\frac{7}{6} + \frac{20}{6} = \frac{27}{6} = \frac{9}{2}$$

$3-y^2 = y+1$

$0 = y^2 + y - 2$

$0 = (y+2)(y-1)$

$y = -2, 1$