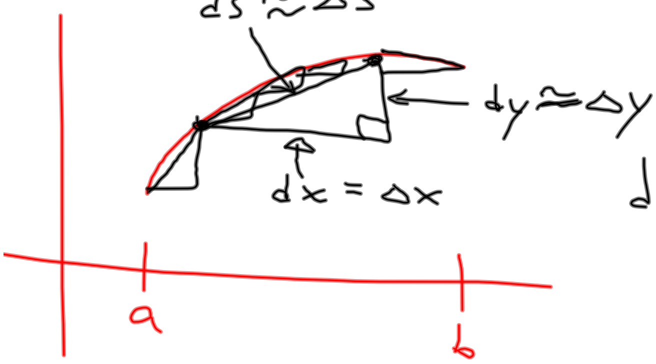
7.4 Surface area and arc length

Arc length formula: $L = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$

$= \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$



$ds = \sqrt{(dx)^2 + (dy)^2}$

$= \frac{dx}{\sqrt{dx^2}} \sqrt{dx^2 + dy^2}$

$= \sqrt{\frac{dx^2 + dy^2}{dx^2}}\,dx$

$= \sqrt{\frac{dx^2}{dx^2} + \frac{dy^2}{dx^2}}\,dx$

$\int ds = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$

Find arc length of $f(x)=\frac{1}{3}x^{3/2}$ on $[0,5]$

$f'(x)=\frac{1}{2}x^{1/2}$

$$L=\int_0^5 \sqrt{1+\left(\frac{1}{2}x^{1/2}\right)^2}\,dx$$

$$=\int_0^5 \sqrt{1+\frac{1}{4}x}\,dx$$

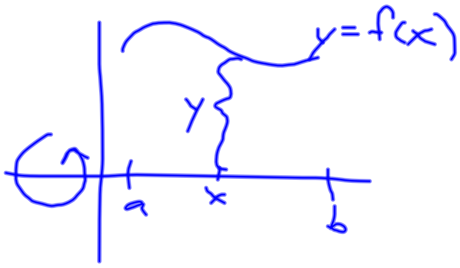
$$=\int_0^5 \left(1+\frac{1}{4}x\right)^{1/2} dx$$

$$=4\cdot\frac{2}{3}\left(1+\frac{1}{4}x\right)^{3/2}\Big|_0^5$$

$$=\frac{8}{3}\left(\frac{27}{8}-1\right)$$

$$\boxed{L=\frac{19}{3}}$$

$$SA = 2\pi \int y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$



Find SA of equation with $y=\sqrt{x}$, $a=0$, $b=2$

$$S=\int_a^b 2\pi y\sqrt{1+y'^2}\,dx$$

$$y=\sqrt{x}=x^{\frac{1}{2}}$$

$$y'=\frac{1}{2\sqrt{x}}$$

$$\sqrt{1+\left(\frac{1}{2\sqrt{x}}\right)^2}=\sqrt{1+\frac{1}{4x}}=\sqrt{\frac{4x+1}{4x}}$$

$$\int_0^2\sqrt{1+(y')^2}\,(2\pi y)\,dx=\int_0^2 2\pi\sqrt{x}\,\frac{\sqrt{4x+1}}{2\sqrt{x}}\,dx$$

$$\pi\int_0^2\sqrt{4x+1}\,dx=\pi\cdot\frac{2}{3}\cdot\frac{1}{4}(4x+1)^{\frac{3}{2}}\Big|_0^2$$

$$=\frac{\pi}{6}(27-1)=\boxed{\frac{13\pi}{3}}$$