7.6 Moments, Centers of Mass, Centroids

Force=mass x acceleration
F=Ma

Moments & Center of Mass for a 1D system:

Let $m_1, m_2, m_3, ..., m_n$ be located at positions $x_1, x_2, x_3, ..., x_n$.

1. The moment about the origin, $M_0=m_1x_1+m_2x_2+m_3x_3+...+m_nx_n$.

2. The center of mass or center of gravity x bar = M_0/m, where $m=m_1+m_2+m_3+...+m_n$ is the total mass of the system.

ex. A 12 cm beam of a mobile has masses 10units, 15 units, 5 units, and 10 units located at one end, 5cm, 9cm, and 12cm from that end. Find the center of mass.

Moments & Center of Mass for a 2D system:

Let $m_1, m_2, ..., m_n$ be located at $(x_1,y_1), (x_2,y_2), ..., (x_n,y_n)$.

1. The moment about the y axis, $M_y = m_1x_1+m_2x_2+...+m_nx_n$.

2. The moment about the x axis, $M_x = m_1y_1+m_2y_2+...+m_ny_n$.

3. The center of mass or center of gravity is $(\bar{x}, \bar{y})$ where $\bar{x} = My/m$ $\bar{y} = Mx/m$, and $m=m_1+m_2+m_3+...+m_n$ is the total mass o the system

ex. A platter holds items of mass $m_1=6$, $m_2=3$, $m_3=2$, and $m_4=9$ located at (3,-2), (0,0), (-5,3), and (4,2). Locate the spot where an acrobat is to place her finger so as to balance the platter.

Moments and Center of mass of a planar lamina

Let f &g be continuous functions that bound a region (planar lamina) of uniform density ρ such that f(x)≥g(x) on [a,b].

1. The moment about the x axis is $M_x=\rho\int_a^b\left[\frac{f(x)+g(x)}{2}\right][f(x)-g(x)]dx$

2. The moment about the y axis is $M_y=\rho\int_a^b x[f(x)-g(x)]dx$

3. The center of mass or center of gravity is $(\bar{x}\ \ \bar{y})$ where $\bar{x}$ =My/m, $\bar{y}$ =Mx/m, and

$$m=\rho\int_a^b [f(x)-g(x)]dx$$

is the total mass of the system.

ex. Find the center of mass of the lamina of uniform density ρ bounded by $y= 9-x^2$ and the x axis.