8.2 Integration by parts

Thm. If u and v are differentiable functions of x, then

$$\int u\,dv = uv - \int v\,du$$

(ultraviolet voodoo)

$\int x e^{x^2} dx \quad u = x^2$

$du = 2x\,dx$

$\int v\,du + \int u\,dv = uv$

$v\,du + u\,dv = \frac{d}{dx} uv$

In general, let u be the first function you find in "LIATE":

Logarithmic ✓
Inverse Trigonometric
Algebraic ✓
Trigonometric
Exponential

Kick back at Starbuck's with a "LIATE".

or, some people prefer "LIPET":

Logarithmic
Inverse Trigonometric
Polynomial
Exponential
Trigonometric

ex. $\int x^2 \ln x \, dx \quad = uv - \int v\,du$

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P

$u = \ln x \qquad dv = x^2$

$du = \frac{1}{x}\,dx \qquad v = \frac{x^3}{3}$

$= \ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x}\,dx$

$= \ln x \cdot \frac{x^3}{3} - \frac{1}{3}\int x^2\,dx$

$= \ln x \cdot \frac{x^3}{3} - \frac{1}{3} \cdot \frac{x^3}{3} + C$

$= \frac{x^3}{3} \cdot \ln(x) - \frac{x^3}{9} + C$

ex. $\underset{I}{\int \arcsin x}\ \underset{A_P}{dx}$ $= \int \underset{I}{\arcsin x} \cdot \underset{A_P}{1\,dx}$

$y = \arcsin x$ ($\sin^{-1} x$)

$\sin y = \frac{x}{1}$

$\cos y \cdot y' = 1$

$y' = \frac{1}{\cos y}$

$y' = \sec y$

$= \frac{1}{\sqrt{1-x^2}}$

$u = \arcsin x$ $\quad dv = 1\,dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$ $\quad v = x$

$= x \arcsin x + \frac{1}{2}\int \frac{-2x\,dx}{\sqrt{1-x^2}}$

$= x\arcsin x + \frac{1}{2}\int \frac{du}{\sqrt{u}}$ $\quad 1 - x^2 = u$, $-2x = du$

$= x\arcsin x + \frac{1}{2}\int u^{-\frac{1}{2}}\,du$

$= x\arcsin x + \frac{1}{\cancel{2}}\cdot \cancel{2}u^{\frac{1}{2}} + C$

$= x\arcsin x + \sqrt{1-x^2} + C$

ex. $\int e^x \sin x \, dx$

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$u = \sin x \qquad dv = e^x dx$

$du = \cos x \, dx \qquad v = e^x$

$= e^x \sin x - \int e^x \cos x \, dx$

$u = \cos x \qquad dv = e^x dx$

$du = -\sin x \, dx \qquad v = e^x$

$\int e^x \sin x \, dx = e^x \sin x - \left[e^x \cos x - \int -e^x \sin x \, dx \right]$

$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$

$+ \int e^x \sin x \, dx \qquad + \int e^x \sin x \, dx$

$2 \int e^x \sin x \, dx = e^x \sin x - e^x \cos x$

$\int e^x \sin x \, dx = \dfrac{e^x \sin x - e^x \cos x}{2} + C$

ex. $\int \sec^3 x \, dx = \int \underbrace{\sec^2 x} \sec x \, dx$

$u = \sec x \qquad dv = \sec^2 x \, dx$

$du = \sec x \tan x \qquad v = \tan x$

$= \sec x \tan x - \int \tan x \cdot \sec x \tan x \, dx$

$= \sec x \tan x - \int \sec x \tan^2 x \, dx$

$= \sec x \tan x - \int \sec x (\sec^2 x - 1) \, dx$

$+\int \sec^3 x \, dx$

$= \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$

$+\int \sec^3 x \, dx$

$2 \int \sec^3 x \, dx = \sec x \tan x + \int \sec x \, dx$

$\frac{2 \int \sec^3 x \, dx}{2} = \frac{\sec x \tan x + \ln|\sec x + \tan x|}{2} + C$

ex. $\int x^2 e^x \, dx$

A or P E

tablular method

	u & dus	dv & vs
+	x^2	e^x
−	$2x$	e^x
+	2	e^x
−	0	e^x

$x^2e^x - 2xe^x + 2e^x + C$

ex. $\int x^2 \sin 4x \, dx$

A or P T

$+$	x^2	$\sin 4x$
$-$	$2x$	$-\frac{1}{4}\cos 4x$
$+$	2	$-\frac{1}{16}\sin 4x$
$-$	0	$\frac{1}{64}\cos 4x$

$\frac{1}{4}\int 4 \sin 4x$

$u = 4x$

$du = 4\,dx$

$$= -\frac{1}{4}x^2\cos 4x + \frac{1}{8}x\sin 4x + \frac{1}{32}\cos 4x + C$$

ex. If $\int f(x)\sin x \; dx = -f(x)\cos x + - \int 3x^2 \cos x \; dx$

What could f(x) be? $u \quad v - \int v \, du$

$v = -\cos x$

$\int du = \int 3x^2$

$u = x^3 + C$