8.3 Trigonometric integrals (A non-AP topic)

Look for both a function and its derivative in the integrand so you can do u substitution.

ex. $\int \tan x \sec^2 x \, dx$

$\int u \, du$

$\frac{u^2}{2} + C$

$= \frac{(\tan x)^2}{2} + C$

$= \frac{\tan^2 x}{2} + C$

ex. $\int \sin^3 x \cos^4 x \, dx$

$= -\int \sin^2 x \cdot \cos^4 x \cdot (-\sin x \, dx)$

$= -\int (1 - \cos^2 x) \cos^4 x \, (-\sin x) dx$ $\quad du \uparrow \quad \cos x = u$

$= -\int (1 - u^2) \cdot u^4 \cdot du$

$= -\int u^4 - u^6 \, du = -\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + C$

$= -\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + C$

ex. $\int \cos^5 x\,dx = \int \cos^2 x \cdot \cos^2 x \cdot \cos x\,dx$

$\cos x\,dx \to du$

$= \int (1-\sin^2 x)(1-\sin^2 x)\cos x\,dx$

$u = \sin x$

$= \int (1-u^2)(1-u^2)\,du$

$= \int 1 - 2u^2 + u^4\,du$

$= u - \frac{2}{3}u^3 + \frac{u^5}{5} + C$

$= \sin x - \frac{2}{3}\sin^3 x + \frac{\sin^5 x}{5} + C$

ex. $\int \sec^4 x \tan^3 x \, dx$

$= \int \sec^2 x \tan^3 x \sec^2 x \, dx$ $\quad du = \sec^2 x \, dx$, $u = \tan x$

$= \int (\tan^2 x + 1) \tan^3 x \sec^2 x \, dx$

$= \int (u^2 + 1) \cdot u^3 \, du = \int u^5 + u^3 \, du$

$= \frac{u^6}{6} + \frac{u^4}{4} + C = \frac{\tan^6 x}{6} + \frac{\tan^4 x}{4} + C$

ex. $\int \sec^4 x \tan^4 x \, dx$

$= \int \sec^2 x \tan^4 x \sec^2 x \, dx$

$= \int (\tan^2 x + 1) \tan^4 x \sec^2 x \, dx$

$= \int (u^2 + 1) \cdot u^4 \, du = \int u^6 + u^4 \, du$

$= \frac{u^7}{7} + \frac{u^5}{5} + C = \frac{\tan^7 x}{7} + \frac{\tan^5 x}{5} + C$

ex. $\int \frac{\sec x}{\tan^2 x}\,dx = \int \frac{\frac{1}{\cos x}}{\frac{\sin^2 x}{\cos^2 x}}\,dx$

$= \int \frac{1}{\cos x} \cdot \frac{\cos^2 x}{\sin^2 x}\,dx = \int \frac{\cos x\,dx}{\sin^2 x}$ ($du = \cos x\,dx$)

$u = \sin x$

$= \int \frac{du}{u^2} = \int u^{-2}\,du = -u^{-1} + C$

$= \frac{-1}{u} + C = \frac{-1}{\sin x} + C = -\csc x + C$

ex. $\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx$

$= \tan x - x + C$